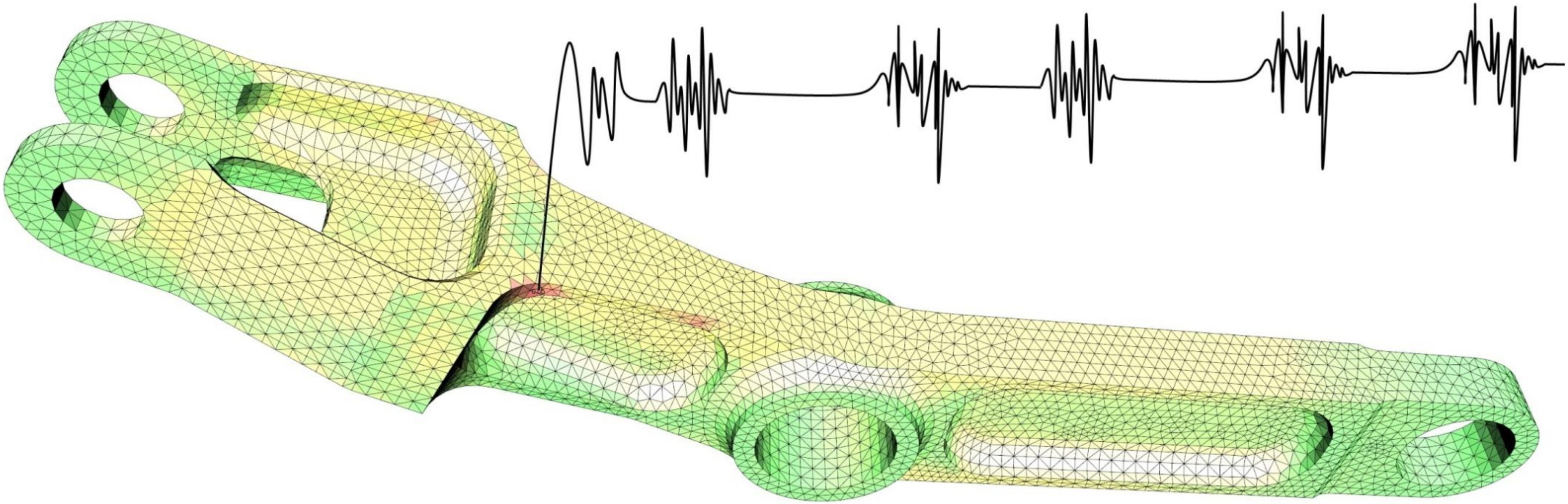


FATIGUE AND DAMAGE TOLERANCE

An insight from the application standpoint

Emphasis to some FEM assisted fatigue analysis aspects

DOMENICO QUARANTA
CHIEF STRESS ENGINEER (TRAINERS DPT) AND FDT SPECIALISTS COORDINATOR – PILATUS AIRCRAFT LTD

OUTLOOK

- About the Lecturer
- Design Criteria Evolution
- Fatigue in Metallic Components
    - Fatigue Models: Uniaxial Stress Based and Strain Based Fatigue
    - Damage Cumulation Rule
    - Cycles Definition and Counting
    - Stress Concentration and Notch Factors

Part 1

- Multiaxial (*FEM assisted*) Fatigue Analysis

Part 2

- Damage Tolerance Analysis (*FEM assisted*)
    - Linear Elastic Fracture Mechanics
    - Stress Intensity Factors
    - Crack Growth
    - Inspection Intervals

Part 3

ABOUT THE LECTURER: DOMENICO QUARANTA

- 1998, MSc Aerospace Engineering at Politecnico di Torino
- *Stress Engineer* - Partenership Alenia-NASA: ISS Node 2 MDPS
- *Stress Engineer* - Aermacchi M346 FWD Fuselage
- *Senior Stress Engineer* - Airbus A380 Trent 900 Nacelles
- *Senior Stress Engineer* - Airbus A380 Main Deck Cargo Door
- *Chief Stress Engineer Trainers/Coordinator Fatigue Specialists* - Pilatus Aircraft (PC-9, PC-9(M), PC-7MkII, PC-21)
- *Lecturer - Fatigue-Damage Tolerance* at Universities of Zürich, Brescia, Parma, Milano, Napoli

DESIGN CRITERIA EVOLUTION

19th Century Fatigue

1953- Comet Accidents

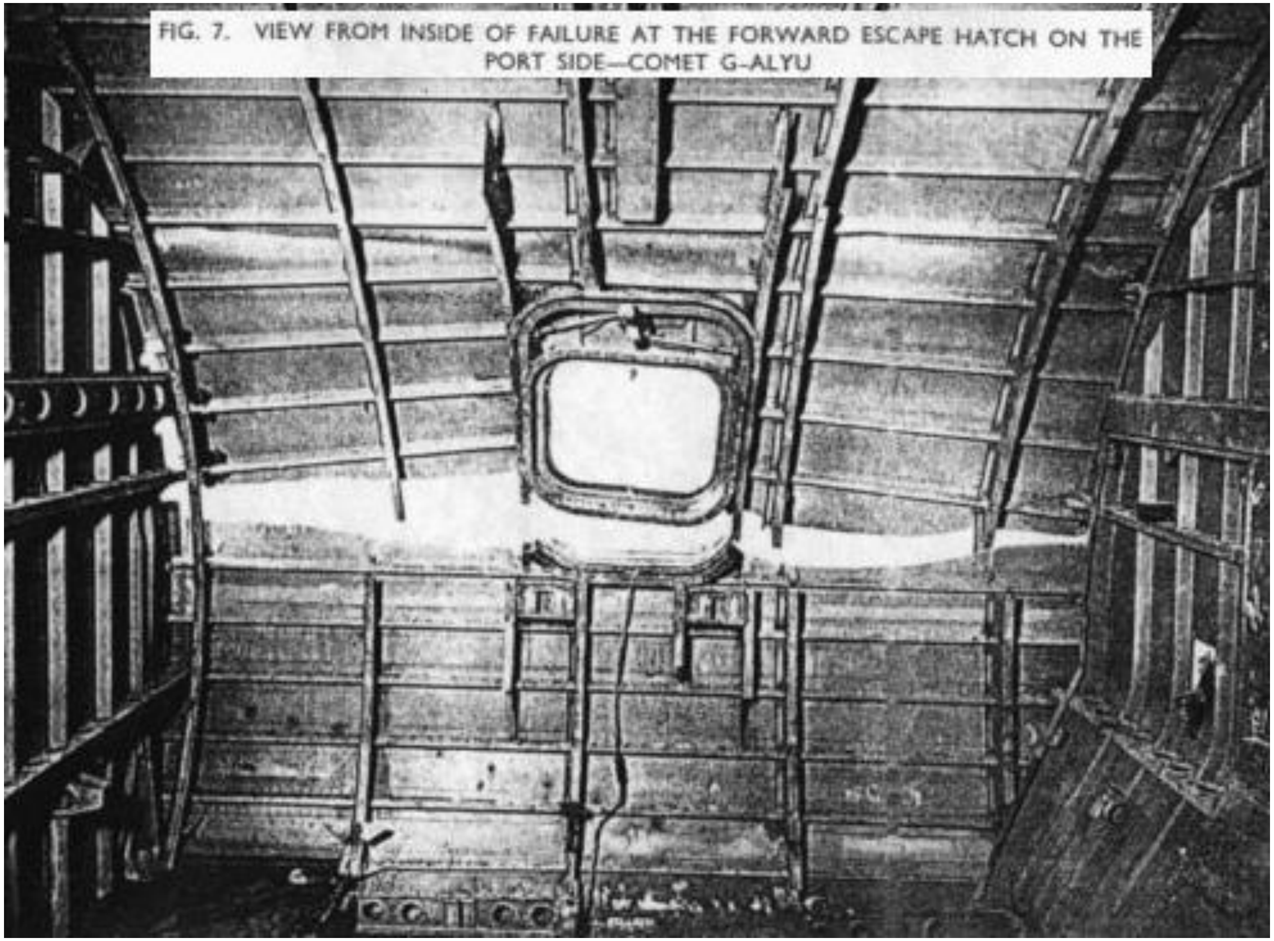
1967- F-111 Accidents

1988 Aloha Flight 243 Accident

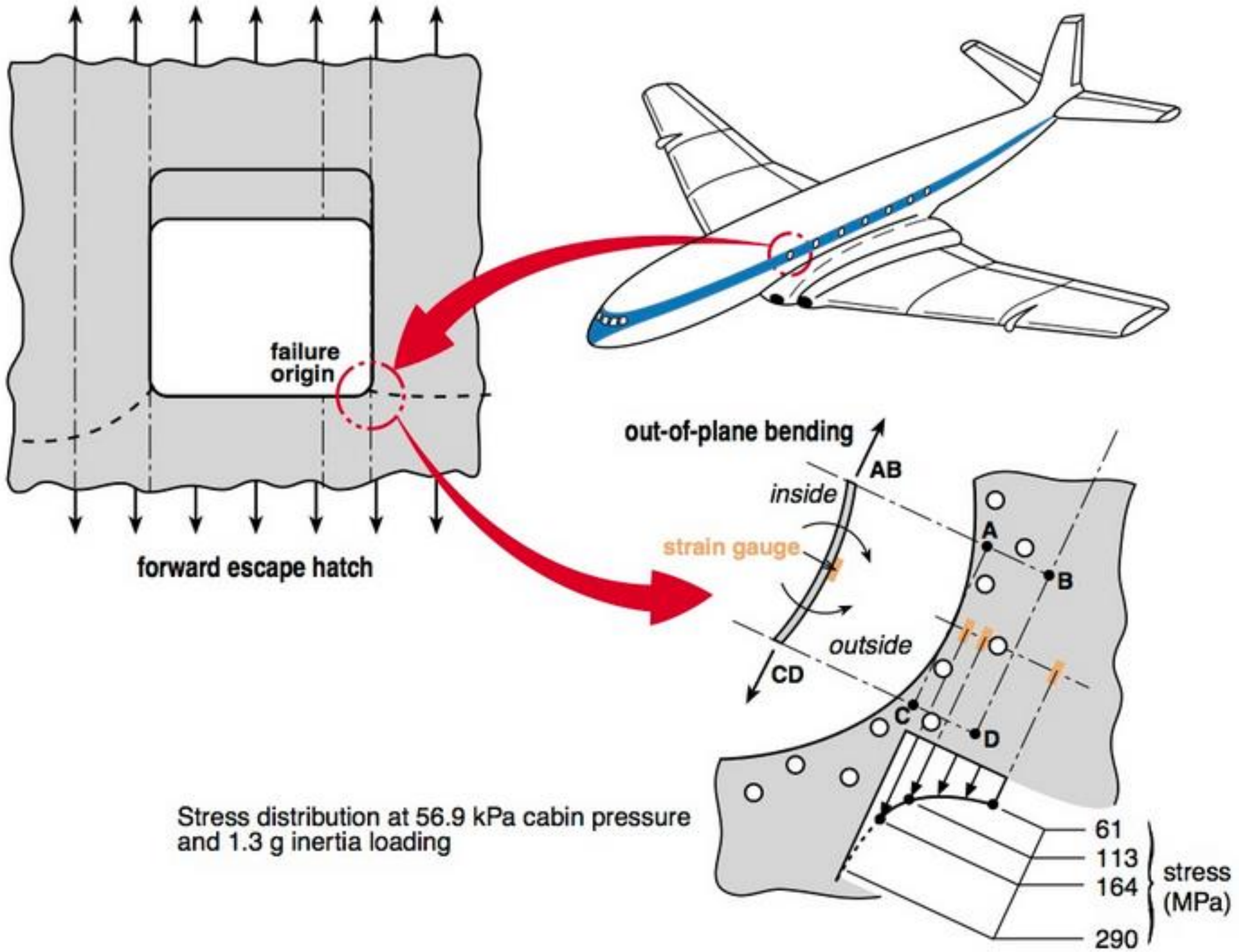
1953 – COMET ACCIDENTS

- 1952 – Enter Service
- From 1953 - Accidents
  - 1954: In flight Failure at 30’000 ft (after 1286 missions)
  - 1954 (after 16 days): In flight Failure at 35’000 ft (after 903 missions)
  - …

1953 – COMET ACCIDENTS



1953 – COMET ACCIDENTS



DESIGN CRITERIA EVOLUTION

Safe Life

Fail Safe

19th Century Fatigue

1953- Comet Accidents

1967- F-111 Accidents

1988 Aloha Flight 243 Accident

SAFE-LIFE DESIGN

- A safe-life design is such that the structure is able to withstand, <u>without catastrophic failure</u>, the repeated loads of variable magnitude expected in service throughout its operational life.
- The structure is retired or replaced at the safe-life to prevent the structure from developing fatigue cracks.
- For a safe-life structure, fatigue failure occurs when a crack is found, therefore the failure concept is related to the presence of a detectable crack.
- Since a safe-life evaluation usually does not include demonstration of crack growth rates or residual strength capability, we assume that the development of a detectable crack may result in catastrophic failure of the structure.
- <u>The calculation process is based on crack initiation stage of the fatigue process.</u>

SAFE-LIFE DESIGN

- By definition, with such concept, no inspections are defined for inadvertent cracks (originated during manufacturing, during installation, during service for accidental events, or pre-existing in the material).

- Should cracks, for any reason (also evaluation errors at design phase), be generated, these, which are ‘unmonitored, will propagate during the Service Life and could lead to catastrophic failures.

FAIL-SAFE DESIGN

- A fail-safe design is a design that retains its required residual strength after the failure or partial failure of a principal structural element.

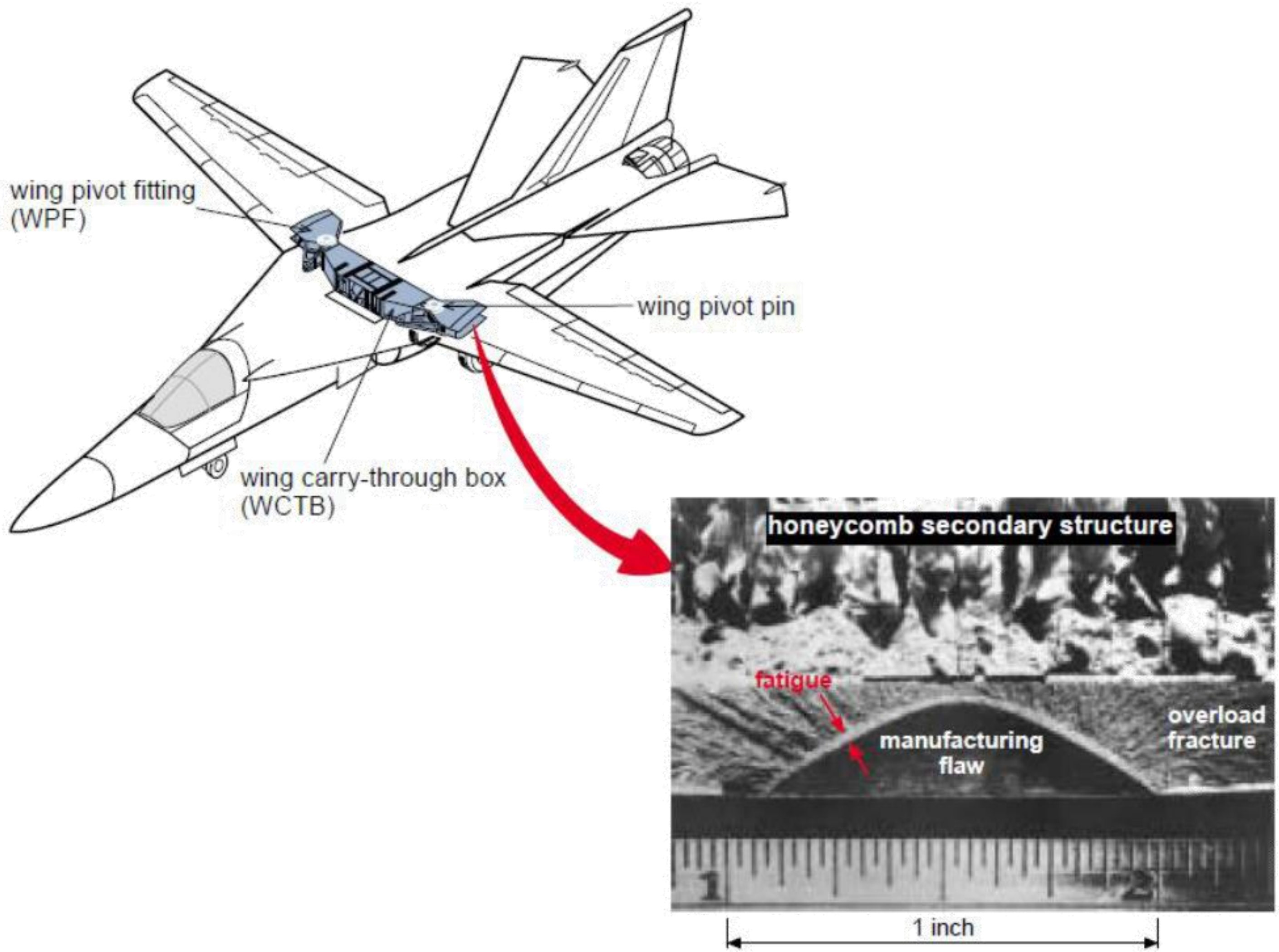
- A fail-safe design typically consists of the fail-safe component or primary structural element, and a redundant or backup structural element.

- A fail-safe design is, therefore, often said to be a redundant design or a multi-load path design.

1967 – F-111 ACCIDENTS

- 1964 – First Flight
- From 1967 – Accidents due to production defects

1967 – F-111 ACCIDENTS



DESIGN CRITERIA EVOLUTION

Damage Tolerance

Safe Life

Fail Safe

19th Century Fatigue

1953- Comet Accidents

1967- F-111 Accidents

1988 Aloha Flight 243 Accident

DAMAGE TOLERANT DESIGN

- Cracks are assumed by definition always existing in the structure: if an inspection doesn’t reveal cracks, it is assumed that cracks are small as the inspection instrument resolution.

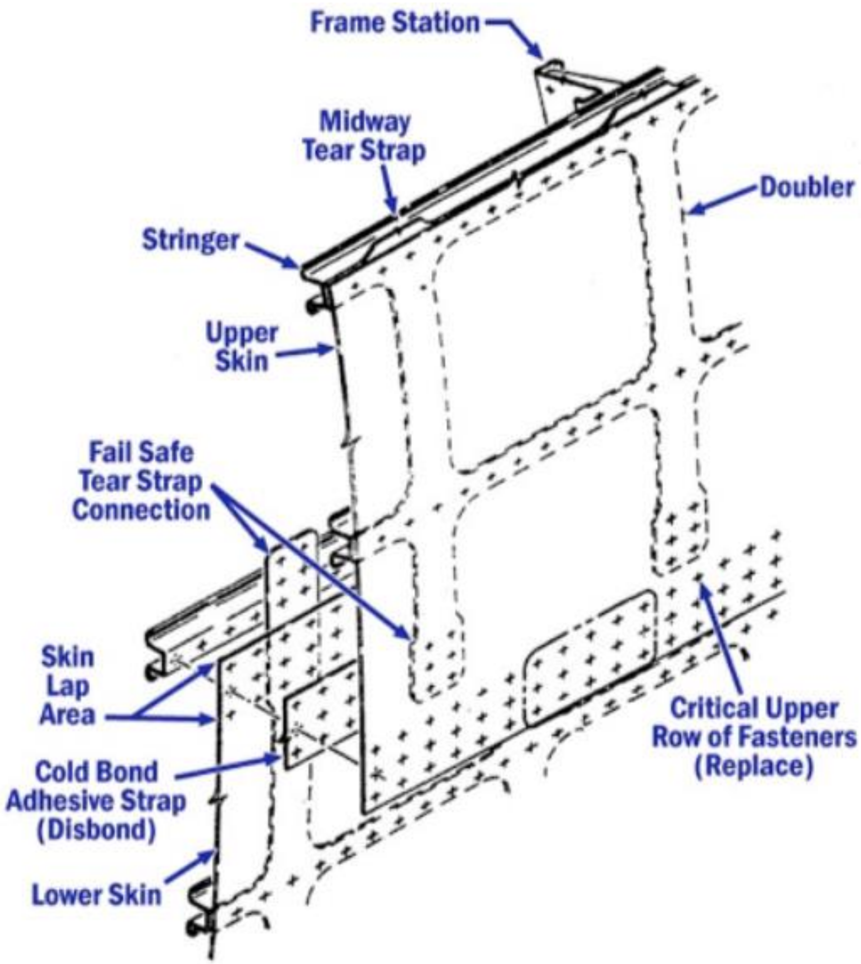
- A structure can retain its required Residual Strength for a period of use after the structure has sustained a given level of fatigue, corrosion, accidental or discrete source damage.

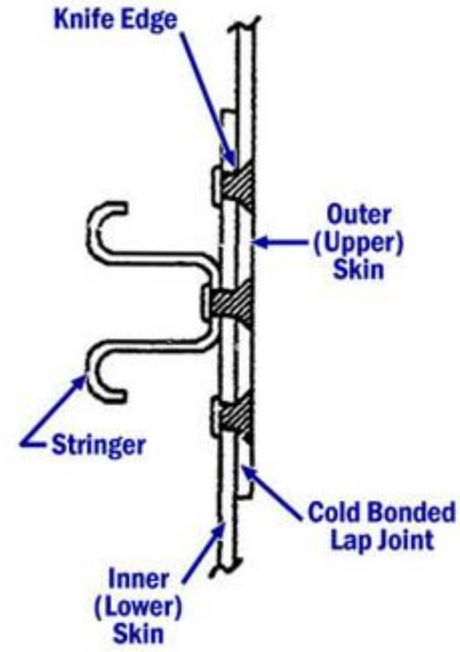
- The concept of inspections is introduced. A Damage Tolerant design is intrinsically safer than the Safe Life, as the structure is regularly inspected for cracks.

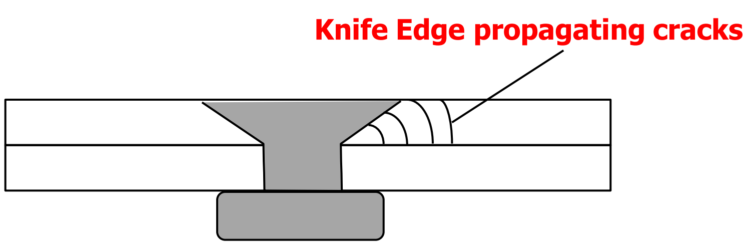
1988 – ALOHA FLIGHT 243 ACCIDENT

1988 – ALOHA FLIGHT 243 ACCIDENT

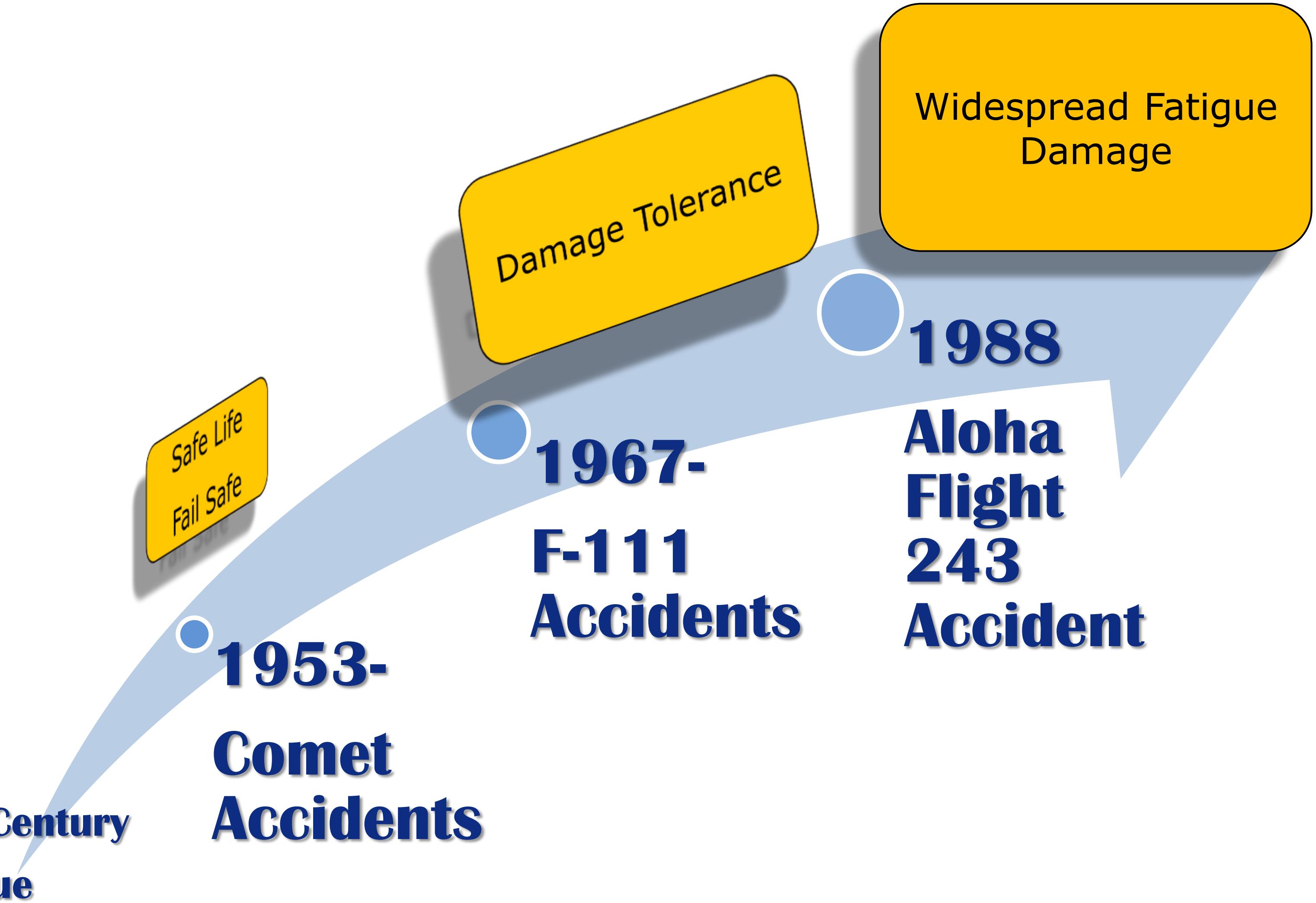
1988 – ALOHA FLIGHT 243 ACCIDENT







DESIGN CRITERIA EVOLUTION



WIDESPREAD FATIGUE DAMAGE (WFD)

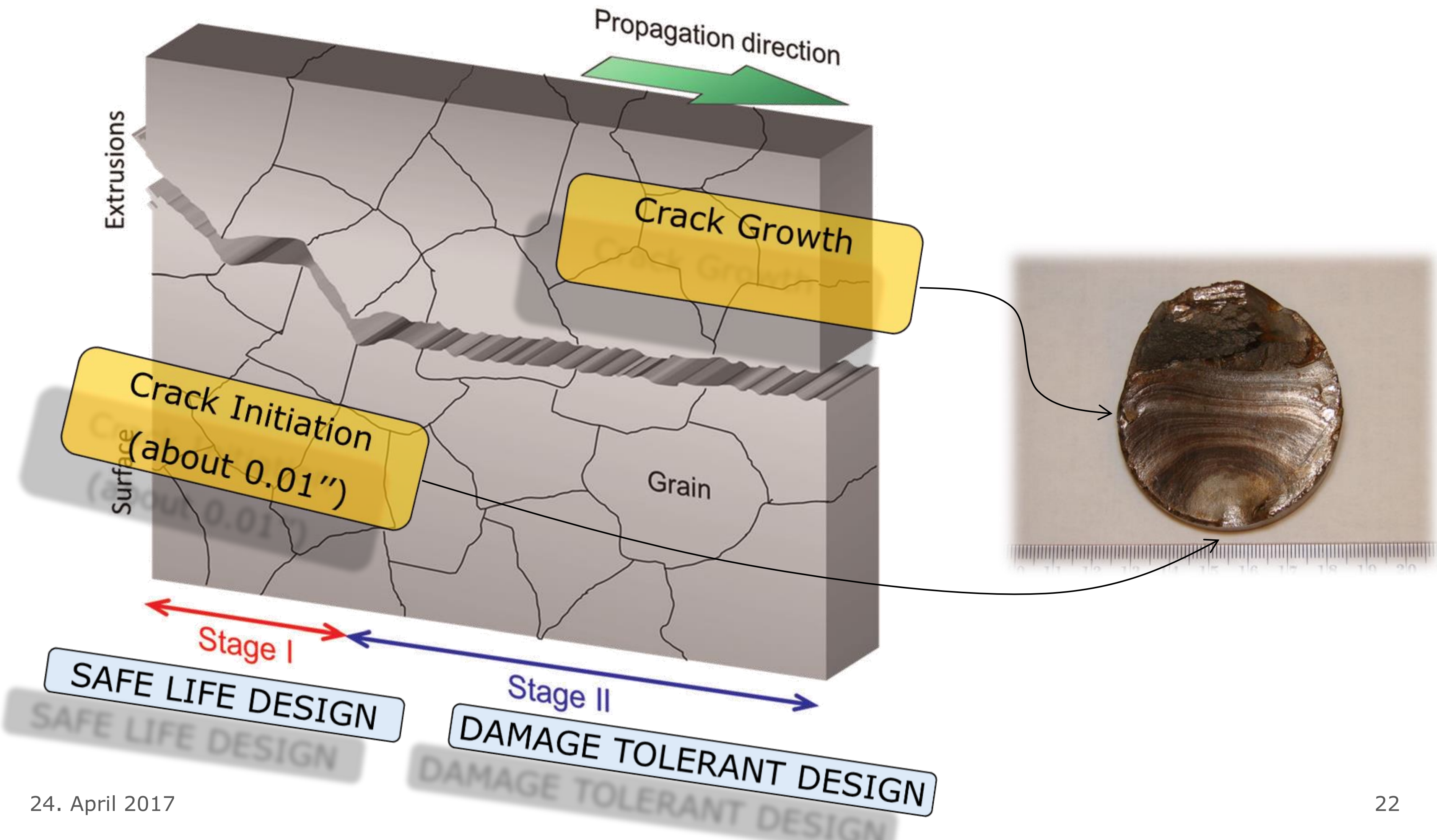
- WFD in a structure is characterized by the simultaneous presence of cracks at multiple points, that are of sufficient size and density such that, the structure will no longer meet its damage tolerance requirement and could fail. For example, small fatigue cracks developed along a row of fastener holes coalesce, this moves to adjacent sites and propagates.

- The objective of a designer is to determine when large numbers of small cracks could degrade the joint strength to an unacceptable level.

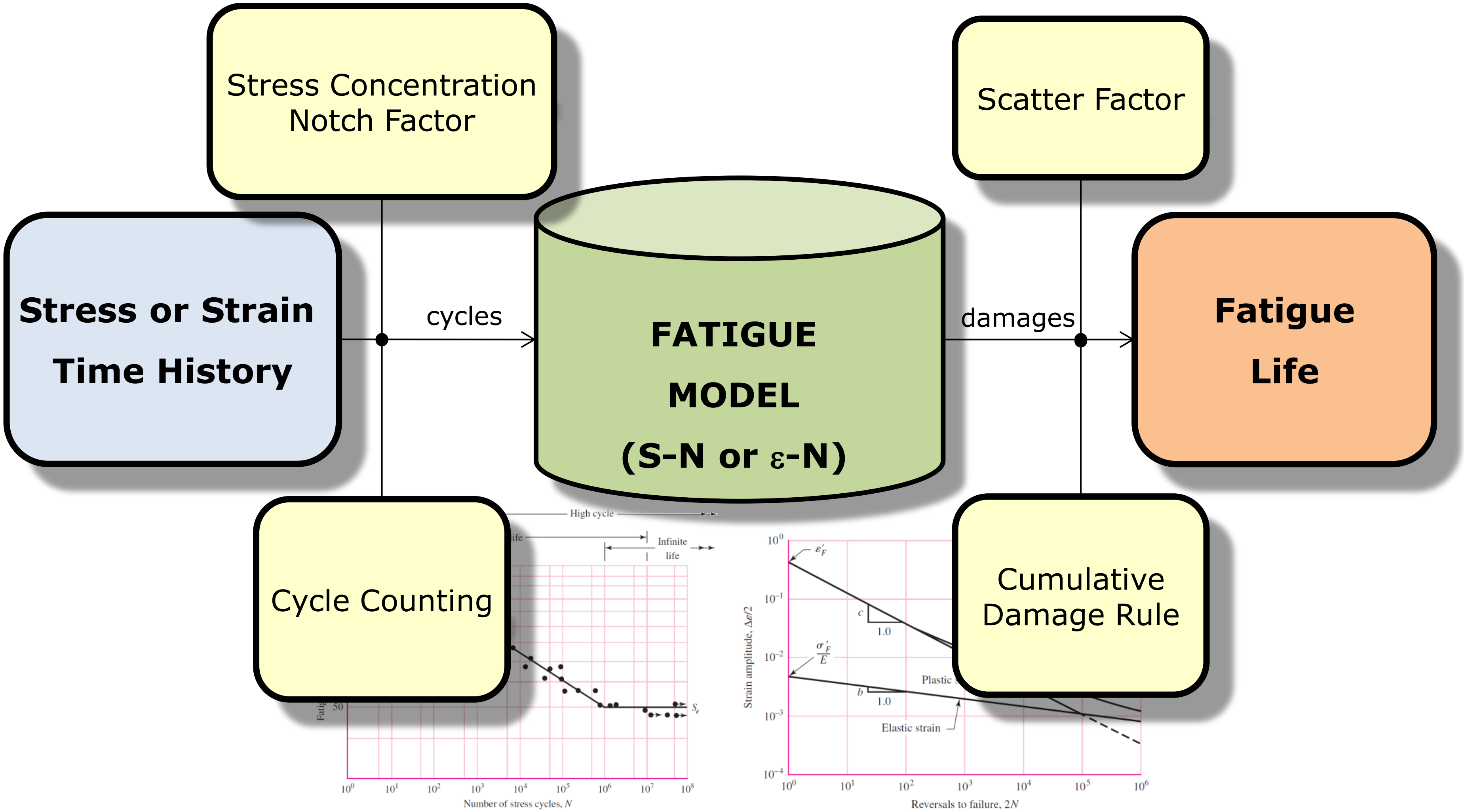
FATIGUE IN METALLIC COMPONENTS

- Fatigue is the progressive and localized structural damage that occurs when a material is subjected to cyclic loading.
- If the cyclic stresses are above a certain threshold value (endurance limit) (for most materials used in lightweight structures the threshold is 0), microscopic cracks nucleate (generally at notches, where there are stress concentrations) after a certain number of cycles.
- Once nucleated, the crack grows up to the critical size, at which the structure suddenly collapses (the remaining section cannot withstand statically the applied cyclic load).
- The fatigue failure occurs at cyclic stress levels which are below the allowable static stress.

FATIGUE IN METALLIC COMPONENTS



FATIGUE IN METALLIC COMPONENTS



FATIGUE MODELS

- There are many fatigue methods that can be used. They belong to two different classes:

  - **Stress based S-N**

    (usually for the High Cycle Fatigue - HCF)

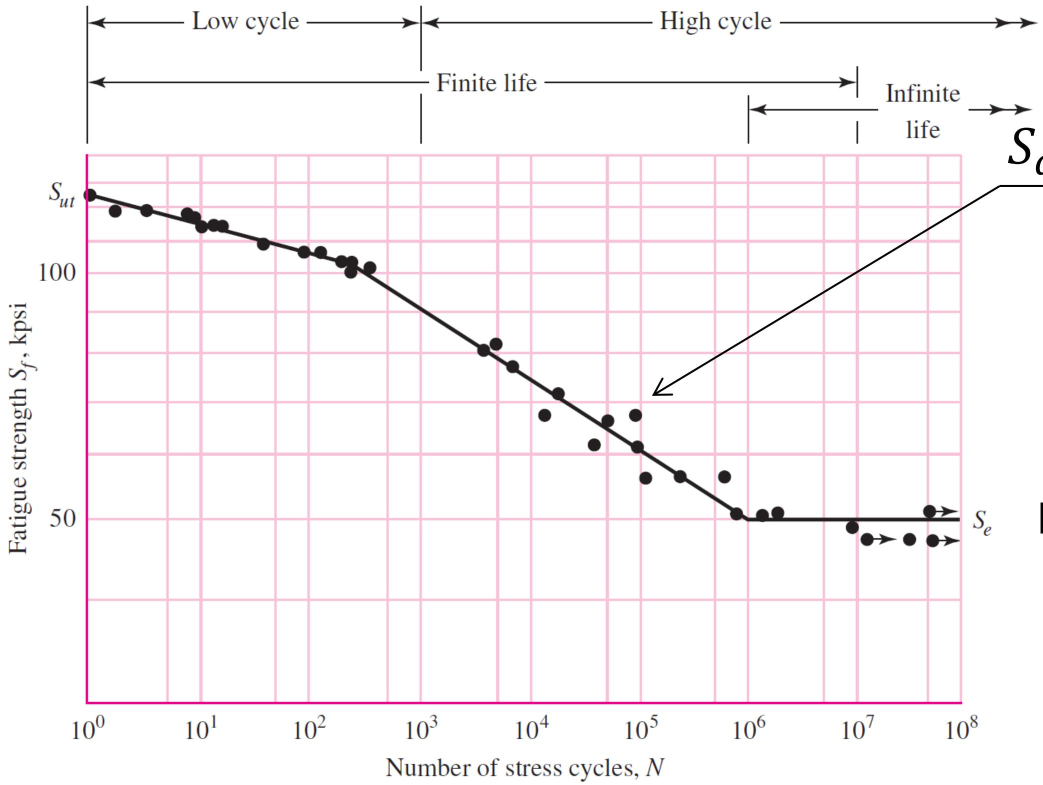
  - **Strain based ε-N**

    (usually for the Low Cycle Fatigue - LCF)

    (This is computationally more involving because elastic-plastic stress and strains have to be calculated)

FATIGUE MODELS

- Stress based: S-N curves

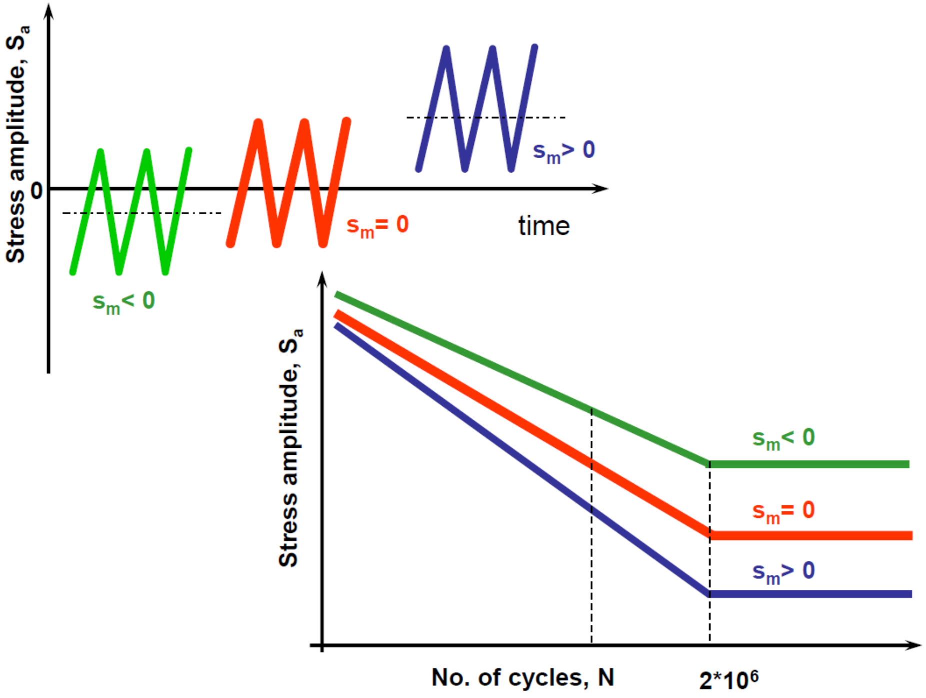


$S_a = C \cdot N^m$ Basquin Equation

Endurance Limit

FATIGUE MODELS

- Stress based: S-N curves
  - S-N curves have to be 'adapted' to the real cases by including:
    - Mean Stress Effects (Goodman, Gerber, Soderberg, Morrow, SWT, Walker, …)
    - Temperature effects
    - Surface conditions
    - Loading modes
    - Size effects
    - Reliability factor
    - …



FATIGUE MODELS

- Stress based: the ‘Metallic Materials Properties Development and Standardization’ (MMPDS) Handbook provides S-N curves for many tested metallic material alloys.

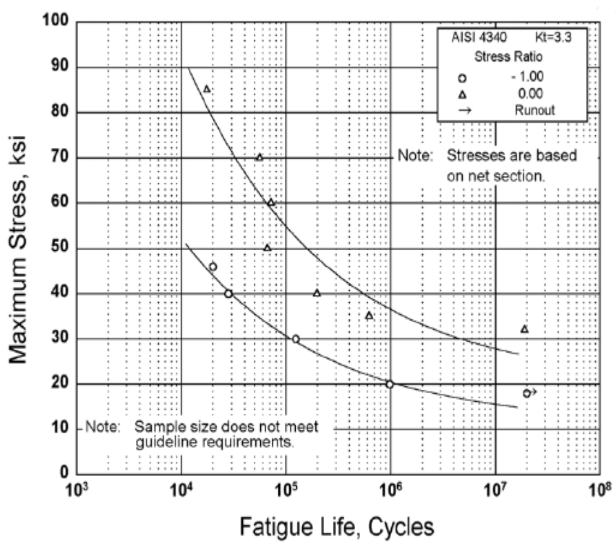


Figure 2.3.1.3.8(b). Best-fit S/N curves for notched, K_t = 3.3, AISI 4340 alloy steel bar, F_{tu} = 125 ksi, longitudinal direction.

Equivalent Stress Equation:
Log N_f = 9.75-3.08 log (S_{eq}-20.0)
$S_{eq} = S_{max} (1-R)^{0.84}$

- The used model is the following:

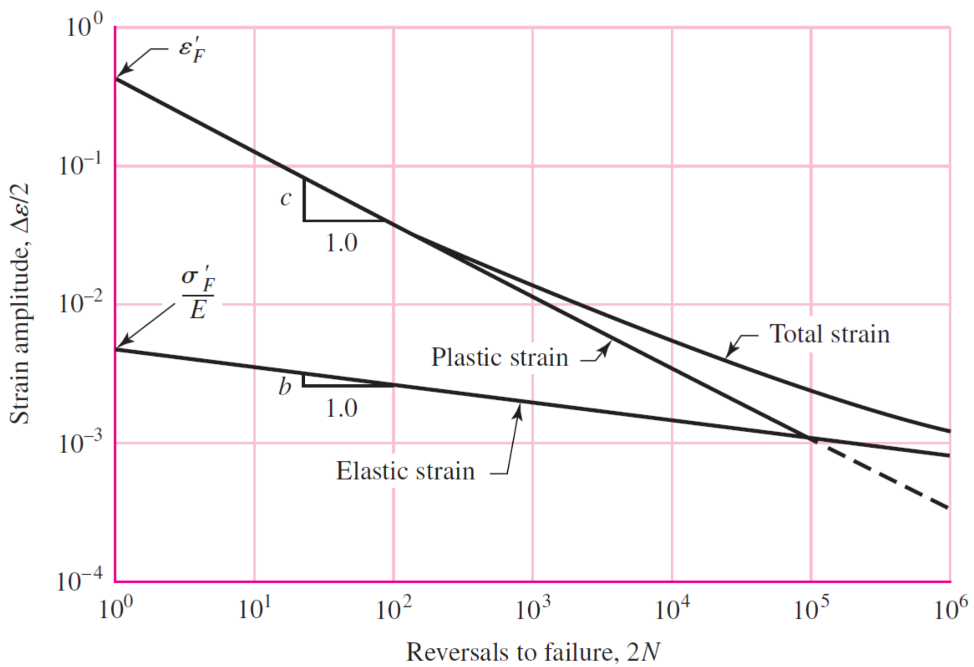
$$Log\ N_f = A - B \cdot Log\left(S_{eq} - C\right)$$

$$S_{eq} = S_{max}(1 - R)^p$$

- Charts provide, for a given material alloy and a given *Kt*, curves for specific stress ratios $R = {}^{S_{min}}/_{S_{max}}$

FATIGUE MODELS

- Strain based: ε-N curves
  - Instead of S-N curves (Basquin equation) the basic model is the Coffin-Manson equation



$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c$$

σ_f' is the Fatigue Strength Coefficient

b is the Fatigue Strength Exponent

ε_f' is the Fatigue Ductility Coefficient

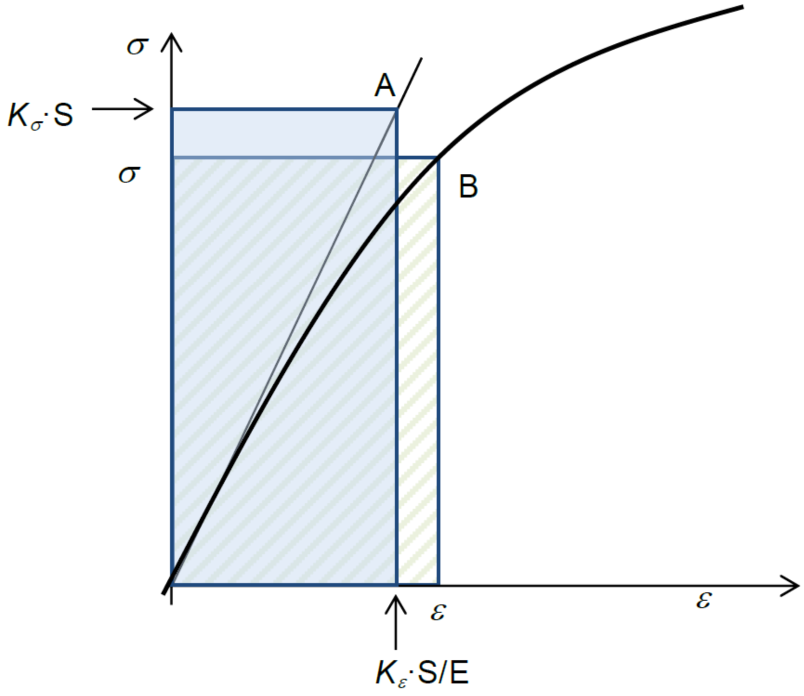
c is the Fatigue Ductility Exponent

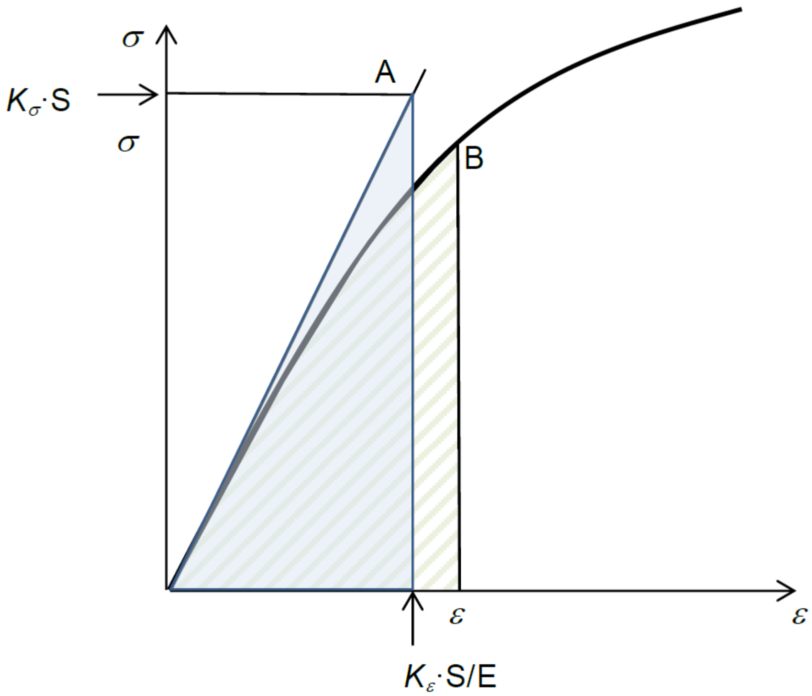
FATIGUE MODELS

- Strain based: ε-N curves
  - Also Coffin-Manson curves have to be 'adapted' to the case under analysis by including:
    - Mean Stress Effects (SWT, Morrow, Manson-Halford, …)
    - Temperature effects
    - Surface conditions
    - ...
  - As said above applying the strain base method implies the additional burden of calculating elastic-plastic stress/strain sequences out of FEM based elastic stress/strain sequences
    - Neuber or Equivalent Strain Energy Density (Glinka) in case of uniaxial fatigue, …
    - … way more complex approaches in case of multiaxial fatigue

FATIGUE MODELS

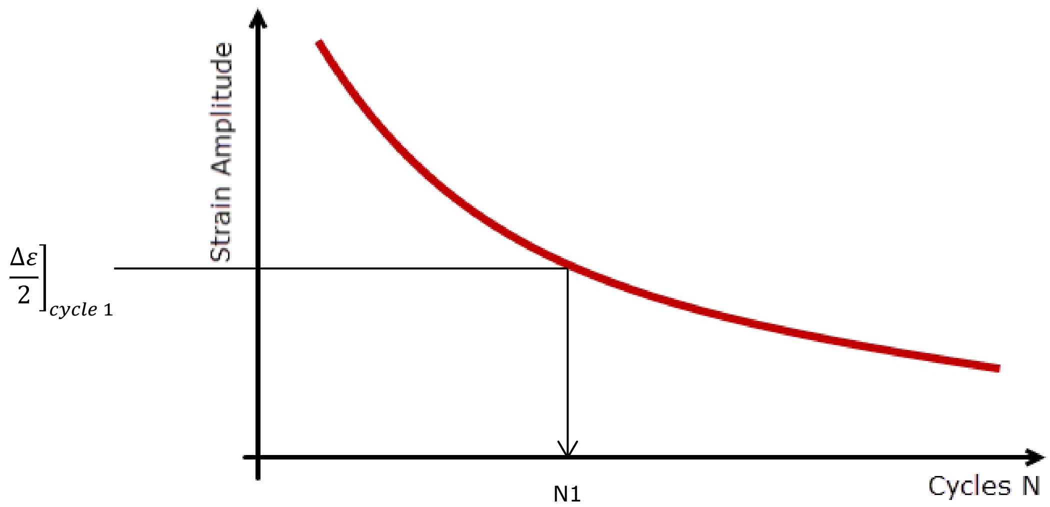
- Strain based: ε-N curves





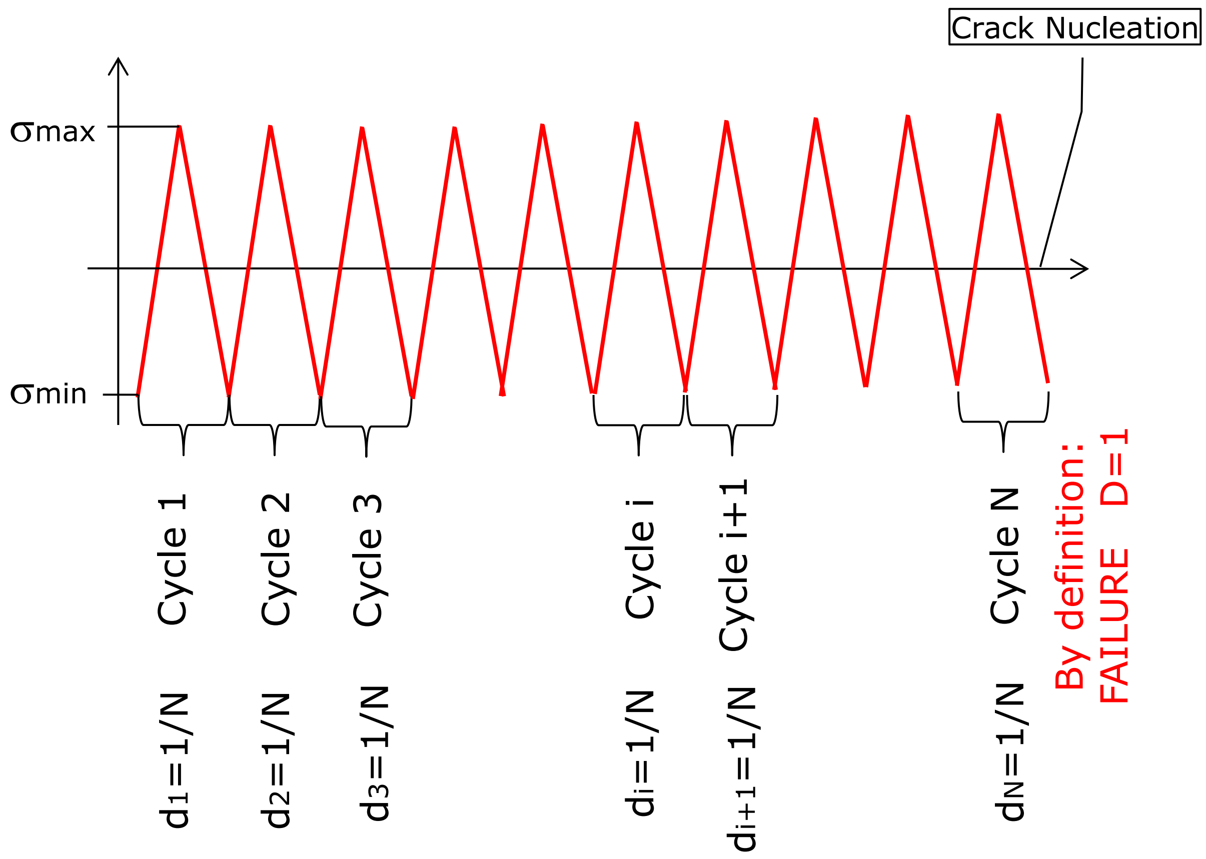
FATIGUE MODELS

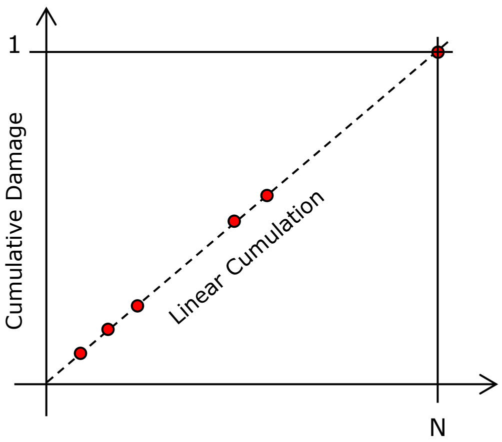
- Whatever the method is (Stress based or Strain based), damage for each cycle is obtained by entering in the modified curve (Basquin or Coffin-Manson) with stress amplitude (for stress based method) or strain amplitude (for strain based method) and extracting a number of cycles which represents the Life related to that specific cycle, i.e. how many of those cycles (constant amplitude) the component survives before a crack is nucleated.



DAMAGE CUMULATION RULE

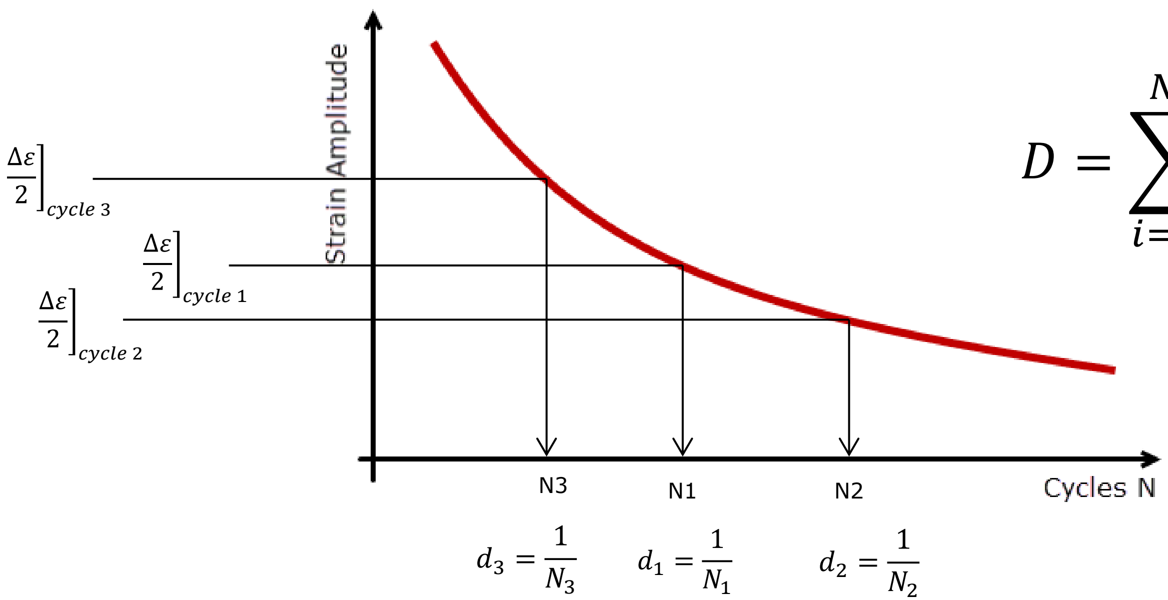
- Miner’s Linear damage cumulation





DAMAGE CUMULATION RULE

- In case of variable amplitude sequences

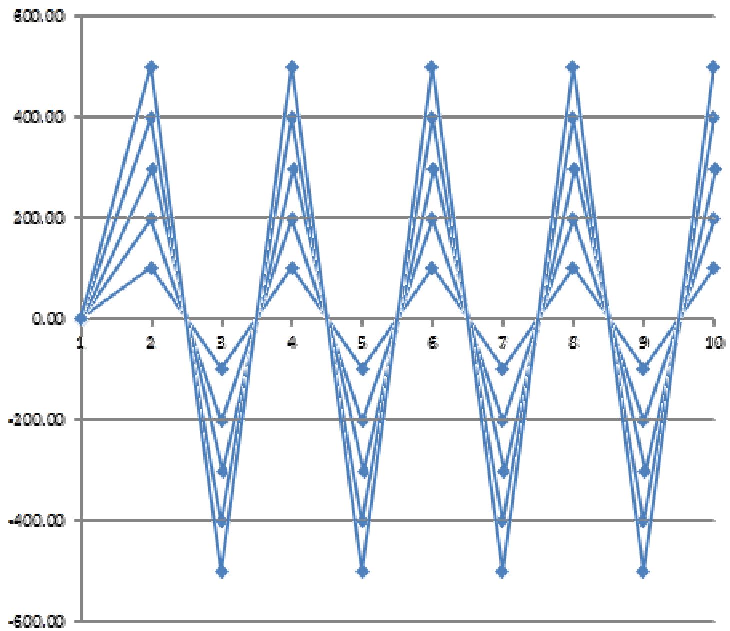


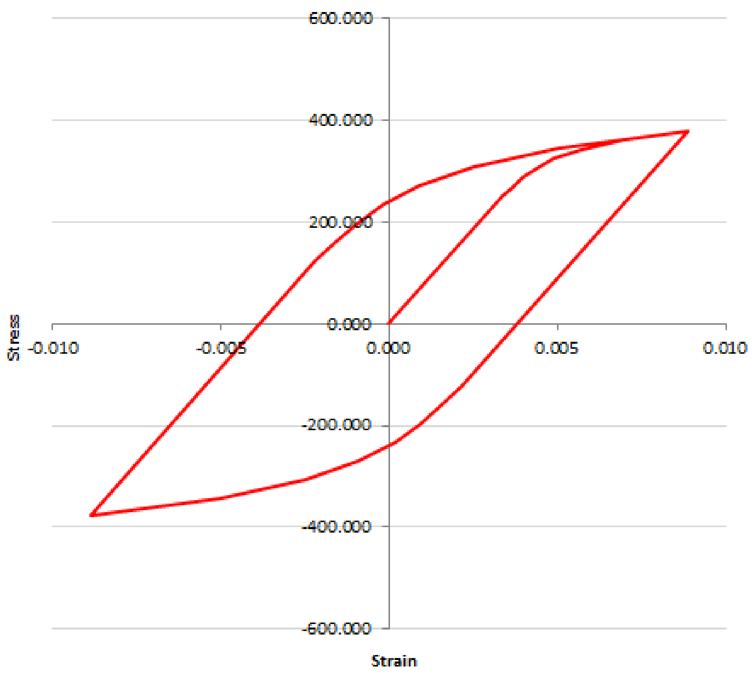
$$D = \sum_{i=1}^{N} d_i \qquad Life = \frac{T}{D}$$

(where T defines the sequence metric)

CYCLES DEFINITION – CYCLE COUNTING

- What is a cycle?
- How a stress cycle is defined?
- What is the physical meaning?





CYCLES DEFINITION – CYCLE COUNTING

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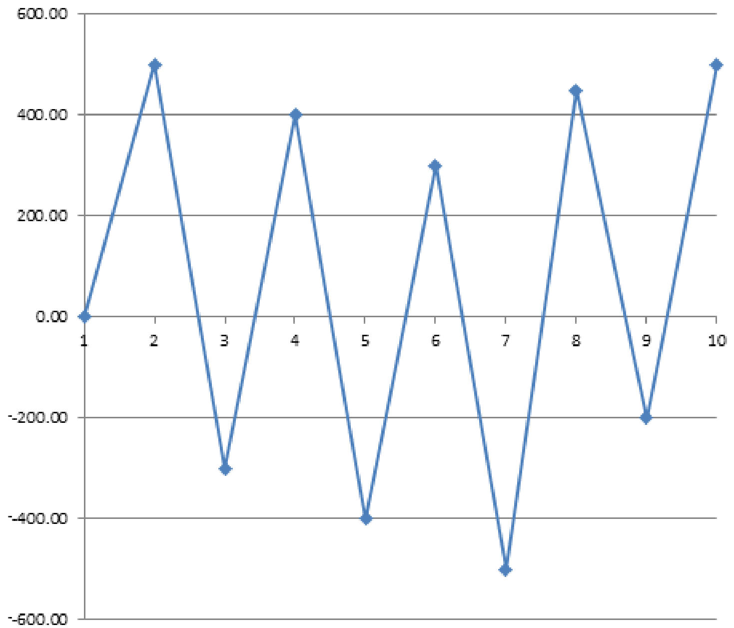
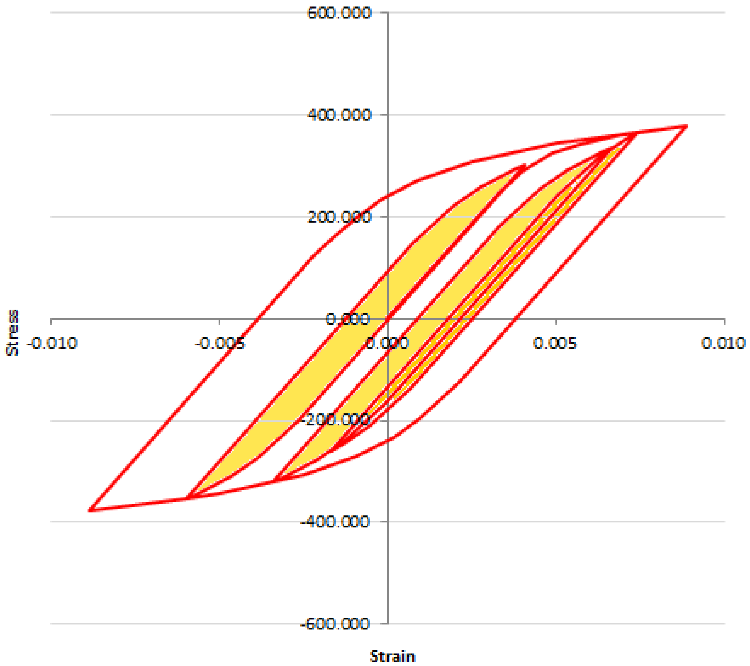
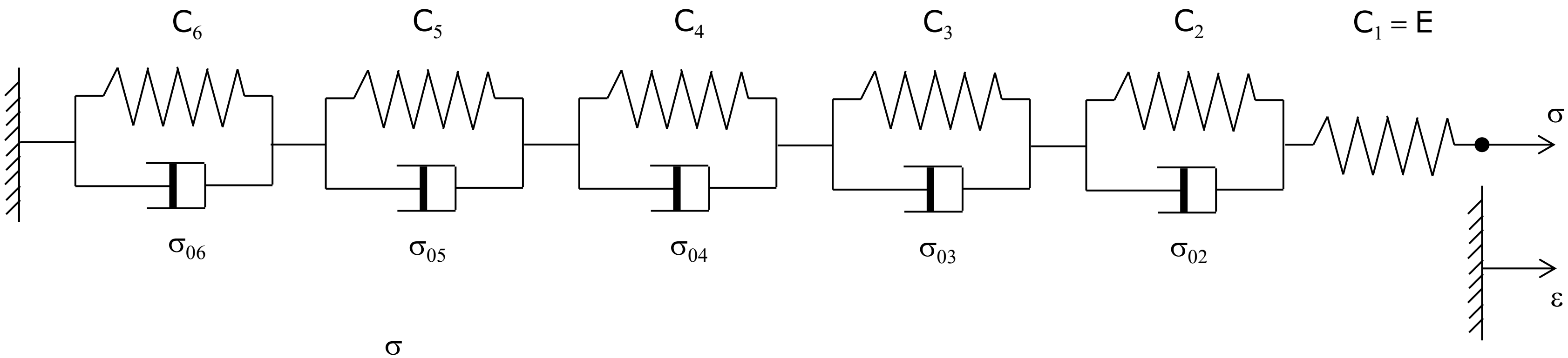
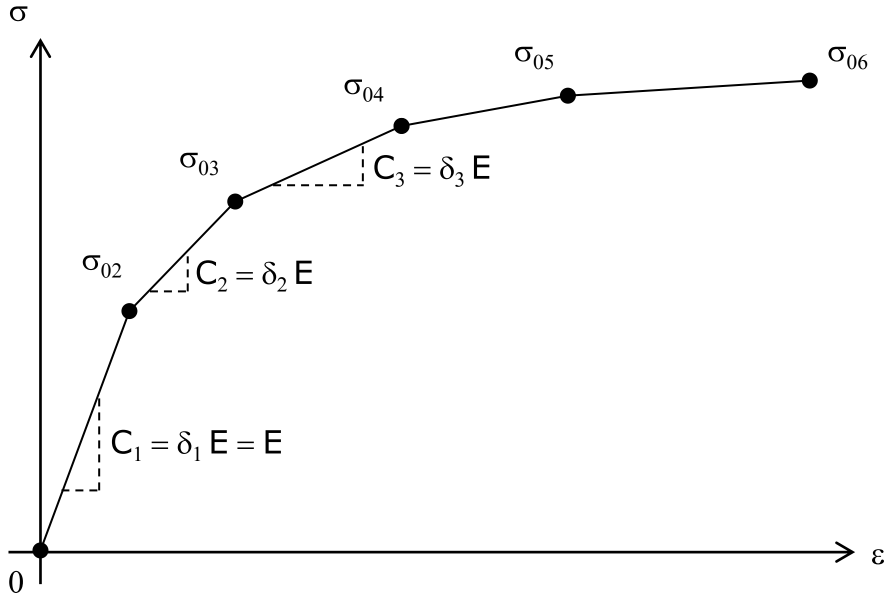




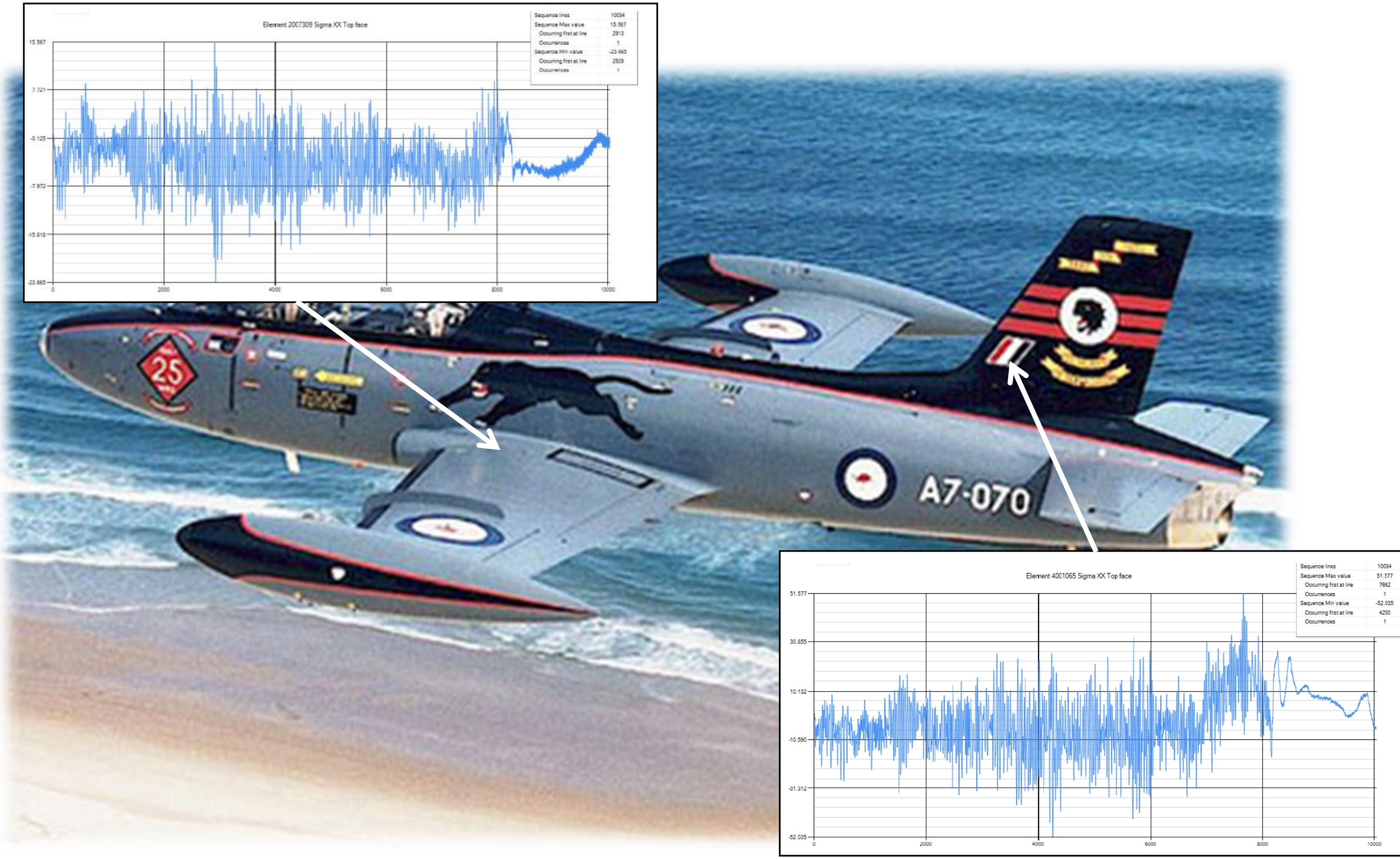
CYCLES DEFINITION – CYCLE COUNTING

- Rheological Model





CYCLES DEFINITION – CYCLE COUNTING



CYCLES DEFINITION – CYCLE COUNTING

- In order to avoid attacking the problem with an ‘incremental approach’ (which is computationally time consuming in case of long time histories), a method/tool is needed to extract cycles (i.e. closed loops) out of a Variable Amplitude spectrum.

- The most popular tool is the Rainflow Cycle Counting (accepted world-wide as the most appropriate for extracting stress/load cycles for fatigue analyses, the algorithm was developed by Endo and Matsuishi in 1968)

- In order to reduce computational time, normally signals are filtered (e.g. Racetrack Filter) before being counted: removal of non-turning points and ‘small’ cycles (i.e. negligibly damaging)

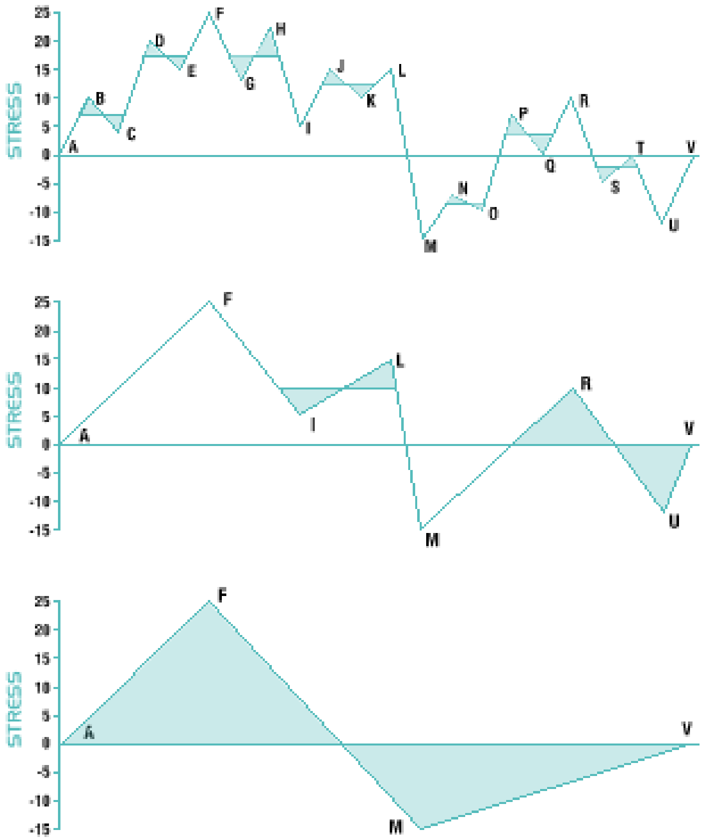
CYCLES DEFINITION – CYCLE COUNTING

- Two consecutive reversal points, *i* and *i-1*, within a sequence represent a peak and valley of a cycle if the conditions applies

$$S_{i-1} < \min(S_{i-2}, S_i)\,;\, S_i > max(S_{i-1}, S_{i+1})$$

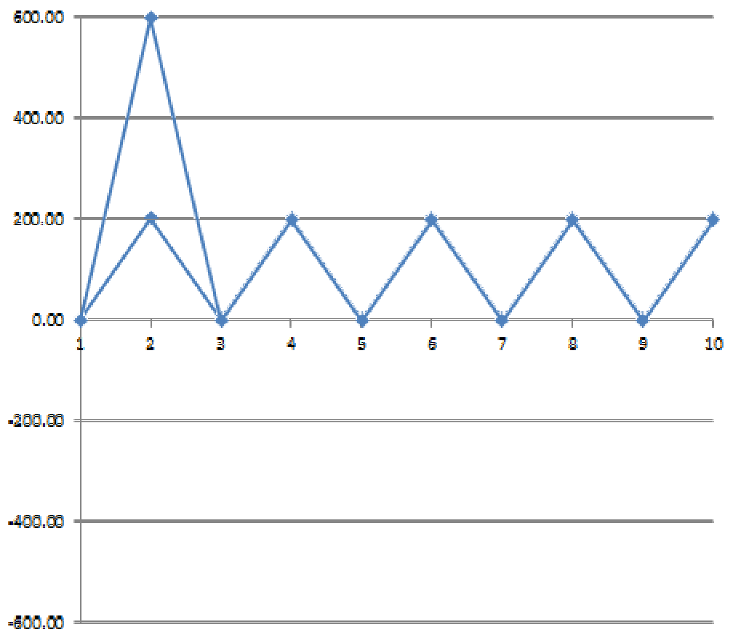
or

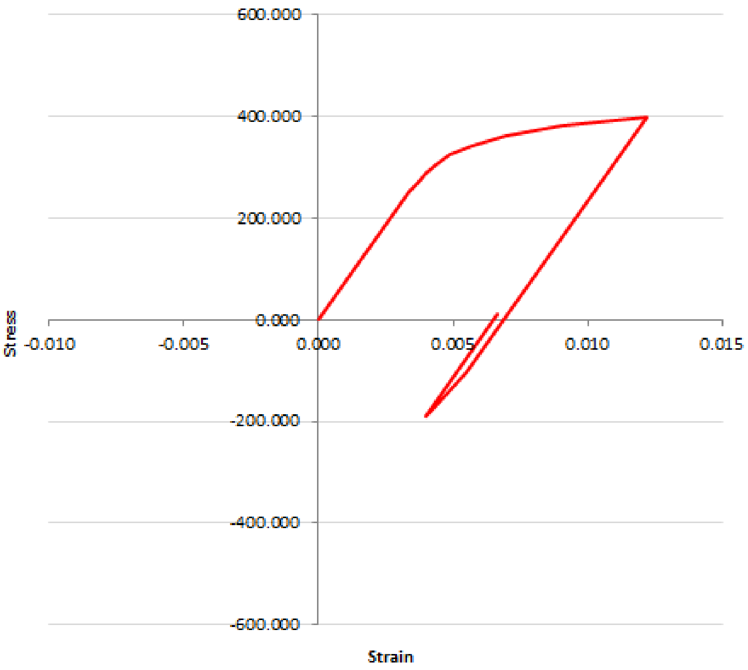
$$S_{i-1} > \max(S_{i-2}, S_i)\,;\, S_i < min(S_{i-1}, S_{i+1})$$



CYCLES DEFINITION – CYCLE COUNTING

- Sequence effects…reminding the Comet…





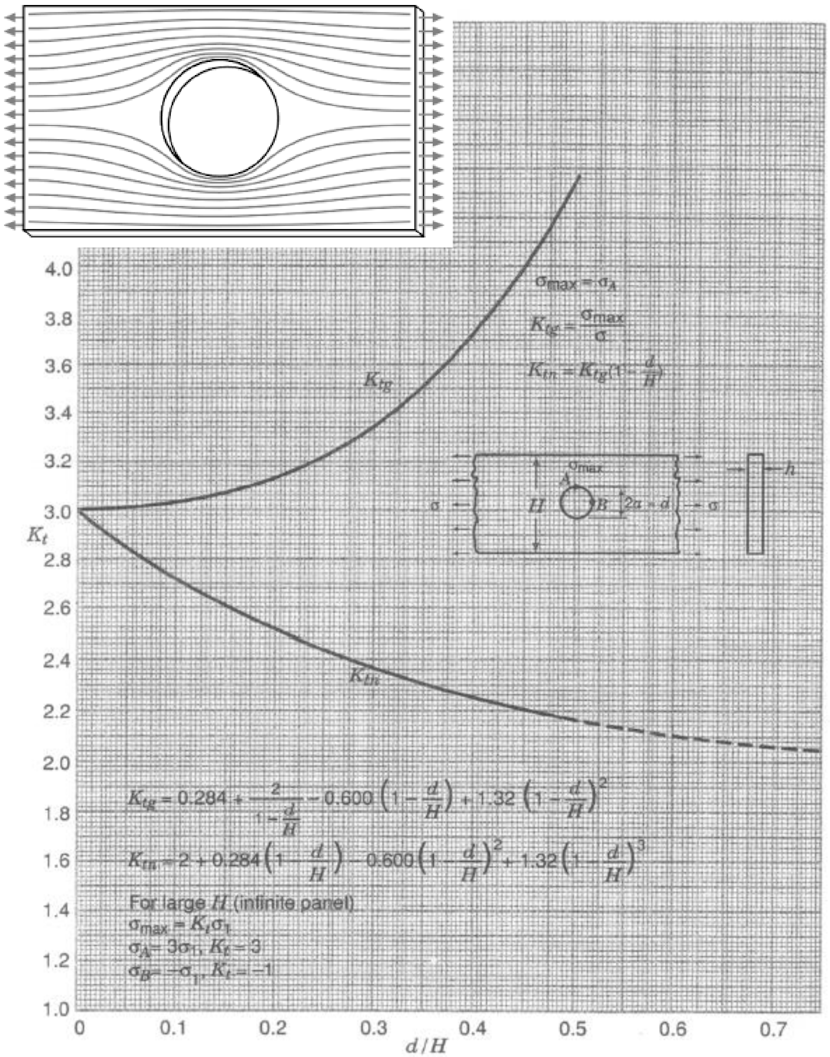
STRESS CONCENTRATION FACTORS

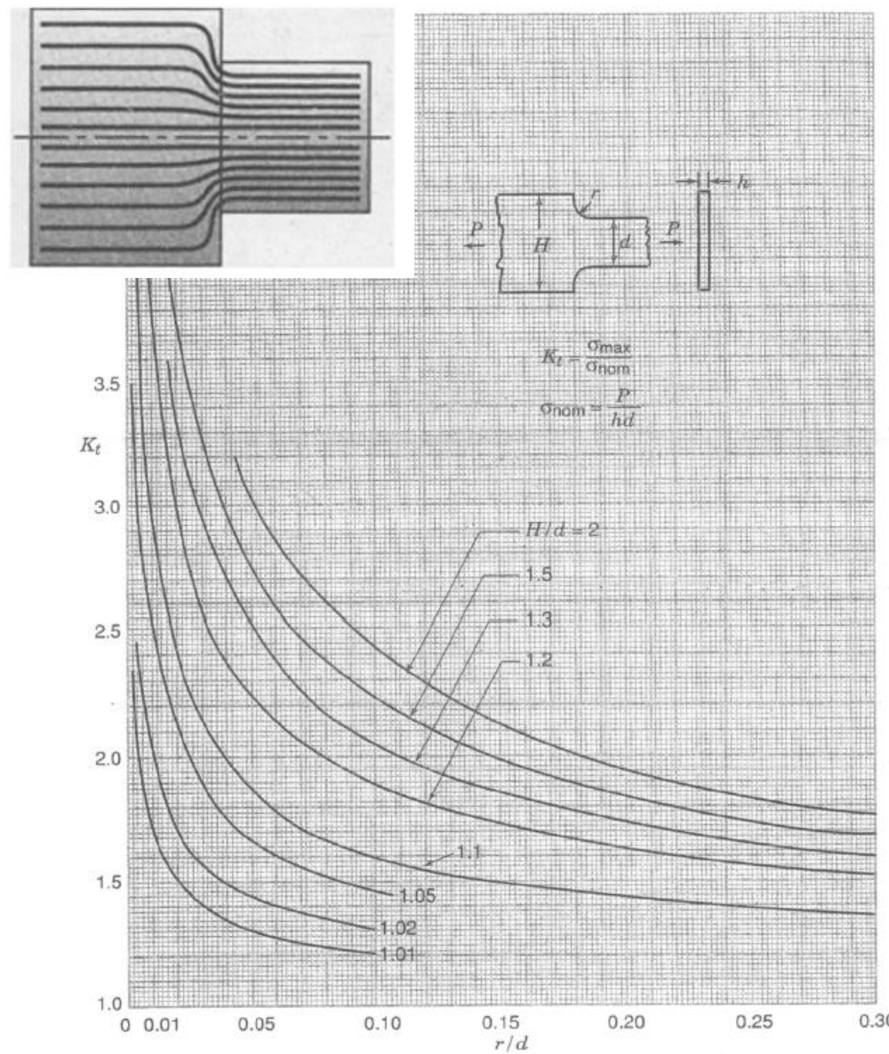
- Geometric discontinuities in a structure such as notches, holes, shoulders, grooves, … are details where stress concentrations occur (stress raisers).
- The Stress Concentration Factor *Kt* is the ratio between the local stress (maximum), at the stress raiser, and the far field (undisturbed), nominal stress

$$K_t = \frac{\sigma_{max}}{\sigma_{nominal}}$$

- Because of higher localized stresses, fatigue failure develops from such details.
- A collection of calculated stress concentration factors is provided by Peterson.

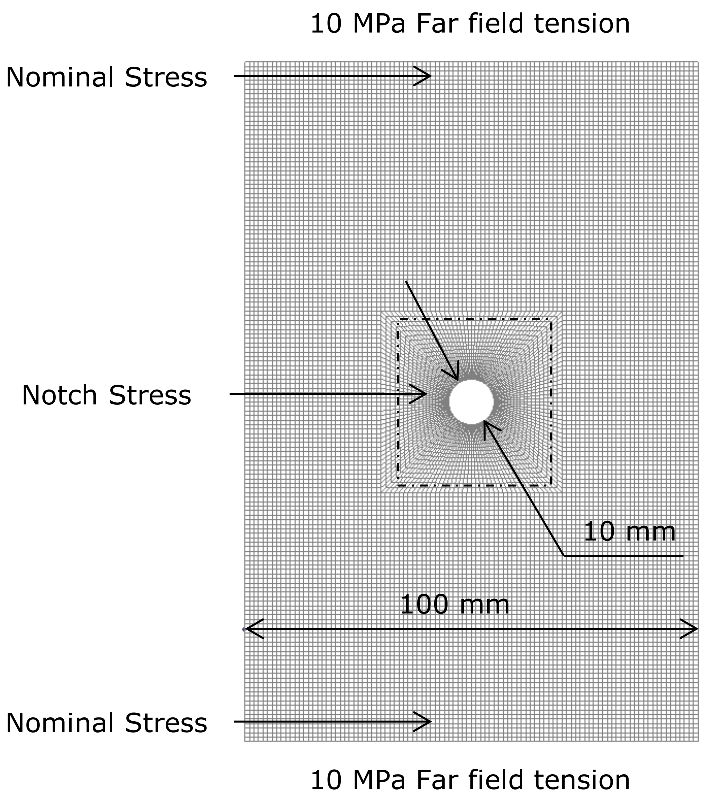
STRESS CONCENTRATION FACTORS

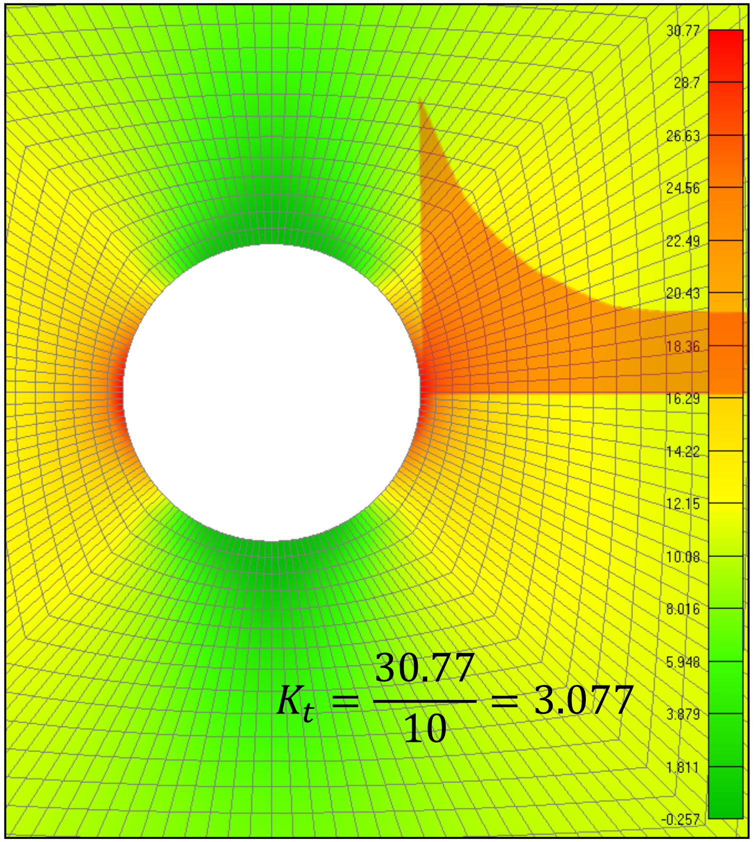




STRESS CONCENTRATION FACTORS

- Today, detailed Finite Element Models can be used to calculate numerically *Kt* for specific geometrical details

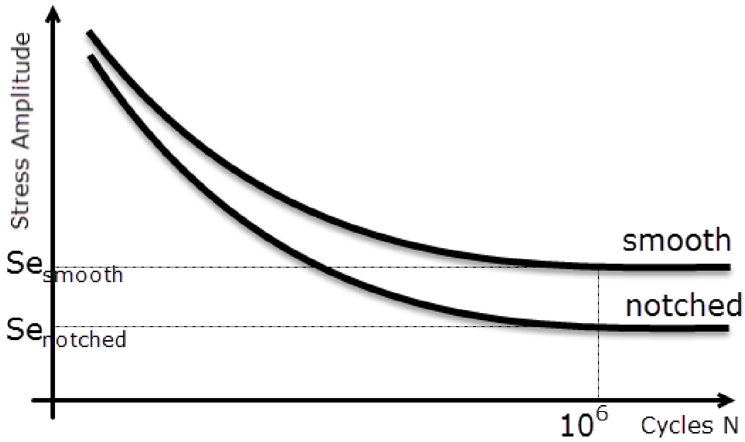




NOTCH FACTORS

- The use of theoretical *Kt*, coming from the assumption of ideal linear elastic materials, is not appropriate in case of applied alternating loads, i.e. fatigue.
- The use of Effective Stress Concentration Factors, or Notch Factors *Kf* is more appropriate in such cases.
- Defined as the Fatigue Strength ratio

$$K_f = \frac{Smooth\ Fatigue\ strength}{Notched\ Fatigue\ strength} = \frac{S_{e,smooth}}{S_{e,notched}}$$



- It is experimentally calculated (at long lives, i.e. >10^6 cycles).

NOTCH FACTORS

- K_f, differently from K_t, is not only geometry and load dependent, but also material dependent:

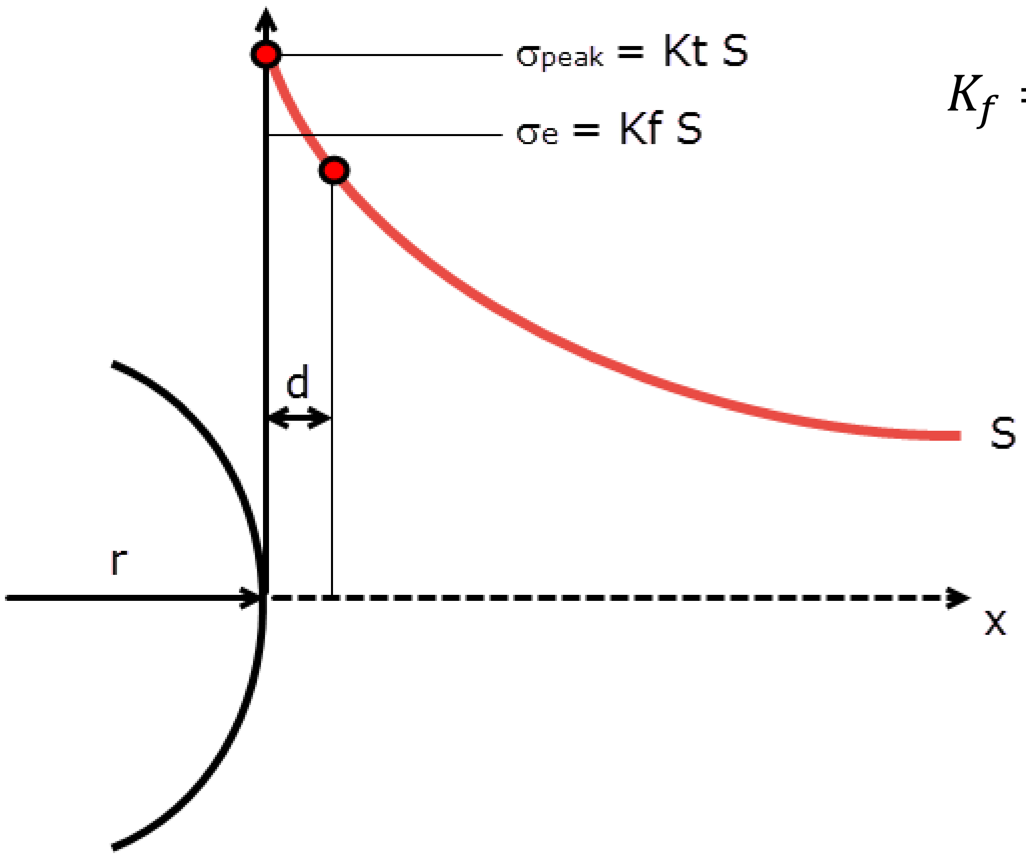
$$K_f = 1 + q(K_t - 1) = 1 + \frac{K_t - 1}{1 + {}^{a}/_{r}} \; < \; K_t$$

r = notch tip radius, a = material constant, q = notch sensitivity factor

- As a material length constant is involved, it implies that <u>two scaled geometries have same K_t but different K_f</u>
- For a given material, the smaller the notch is (small r), the smaller the notch sensitivity is

NOTCH FACTORS

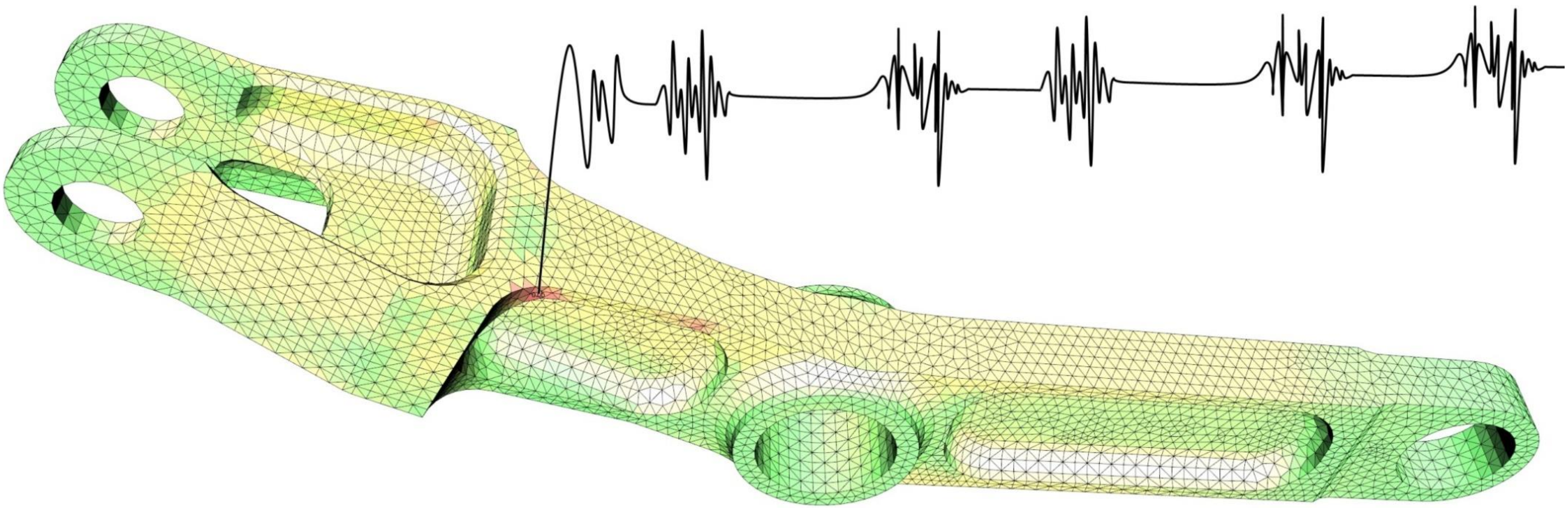
- A way to interpret the notch factor (from Dowling).



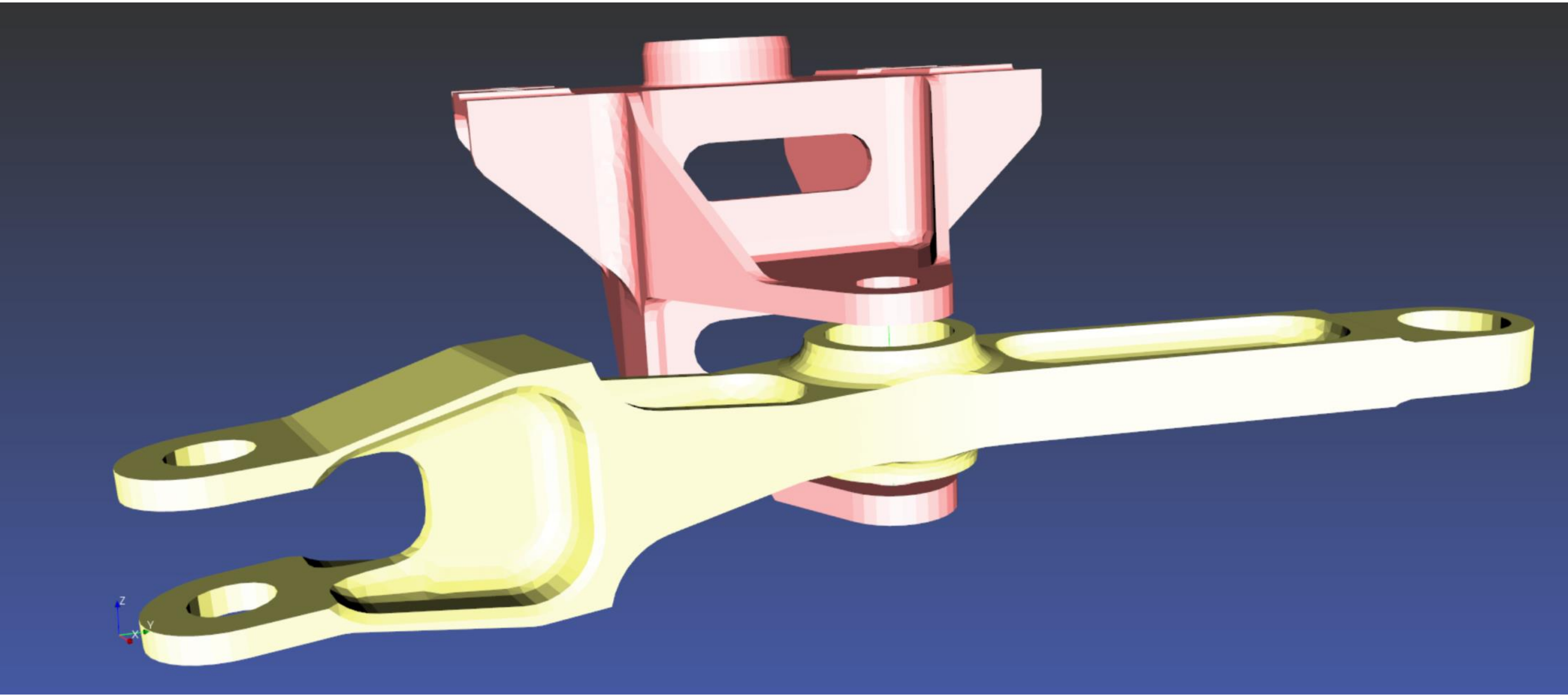
$$K_f = \frac{Stress\ average\ out\ to\ x = d}{S} = \frac{\sigma_e}{S}$$

- The stress controlling initiation of fatigue damage IS NOT the highest stress at the notch surface (x=0), but rather the somewhat lower value that is average out to a distance x=d.

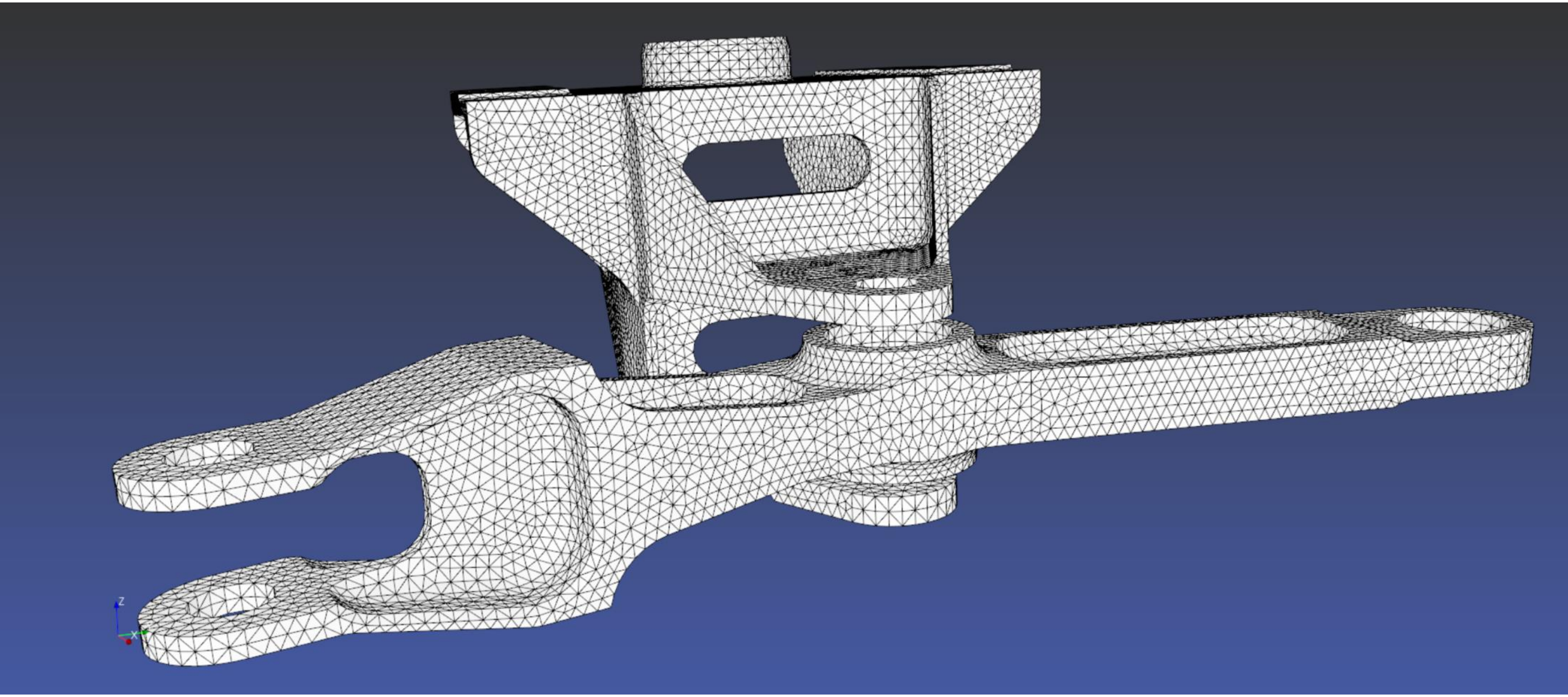
Part 2



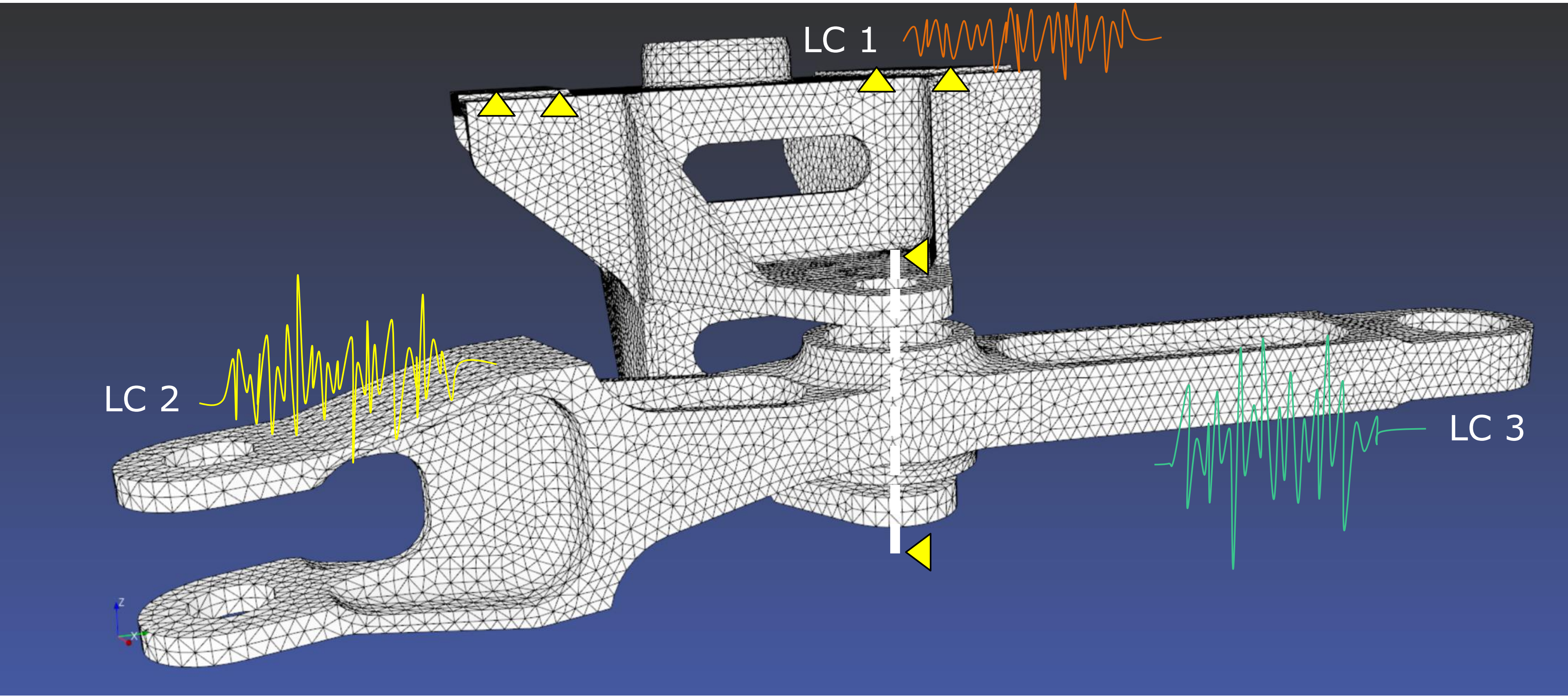
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS



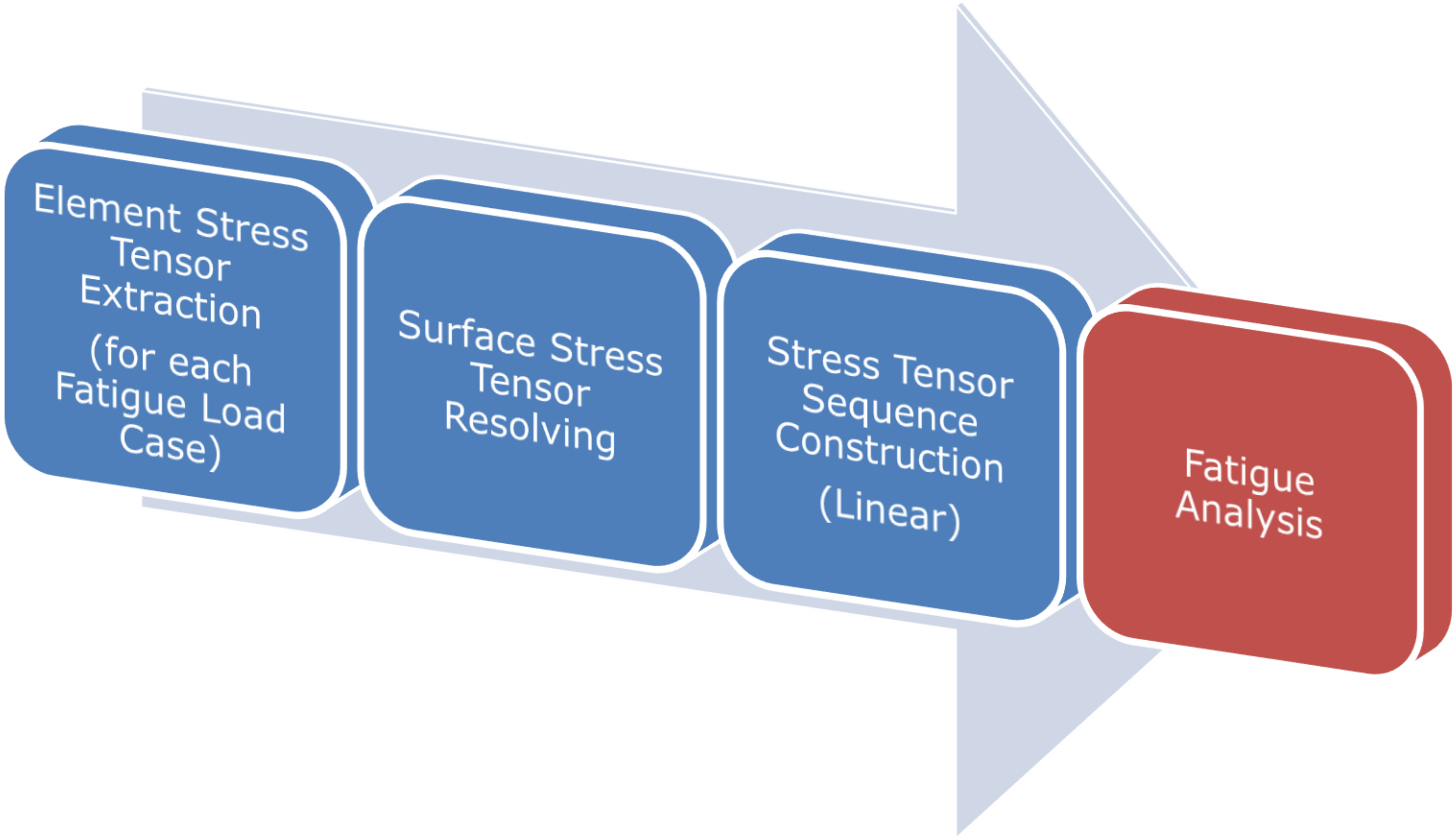
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS



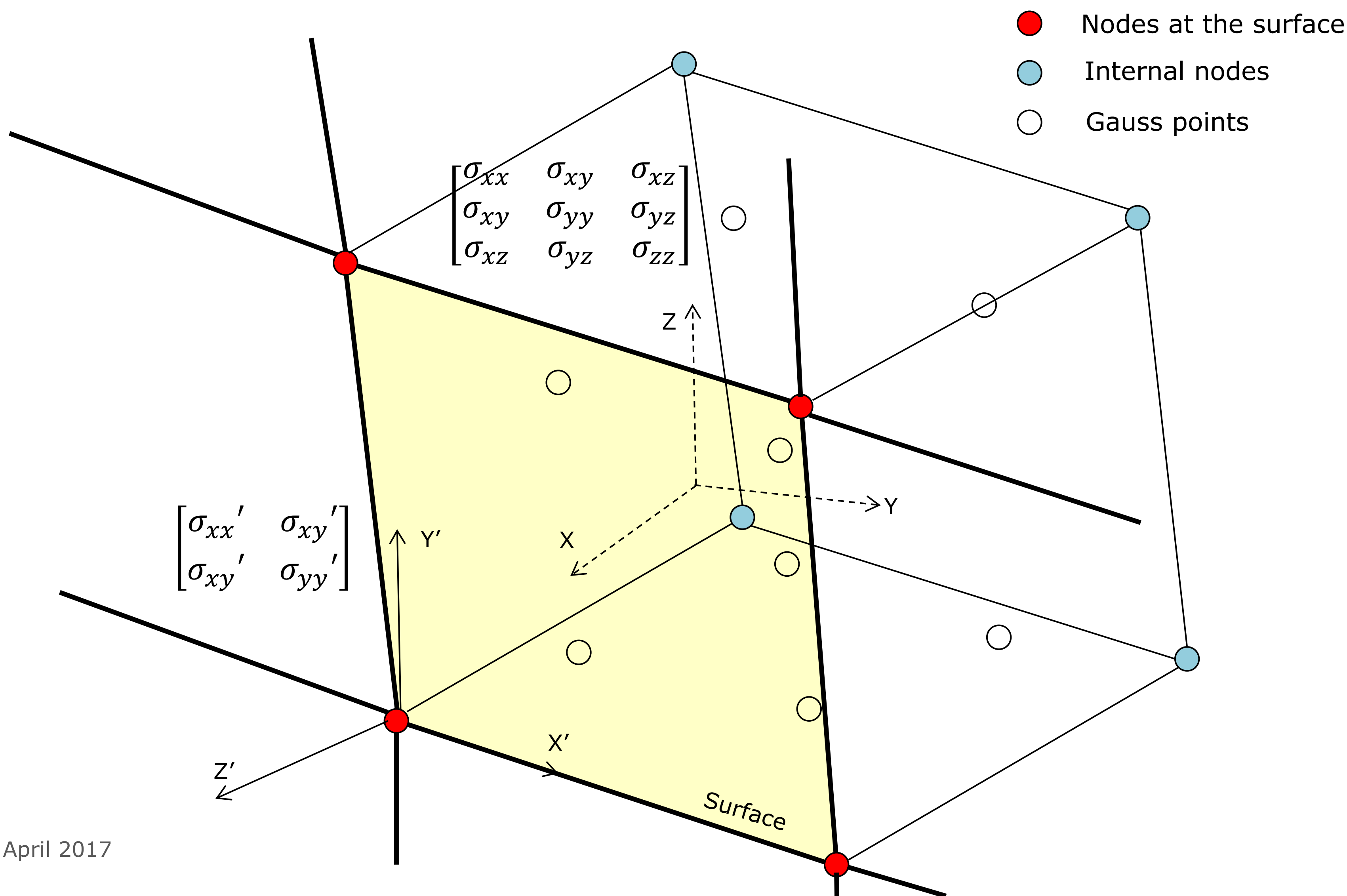
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS



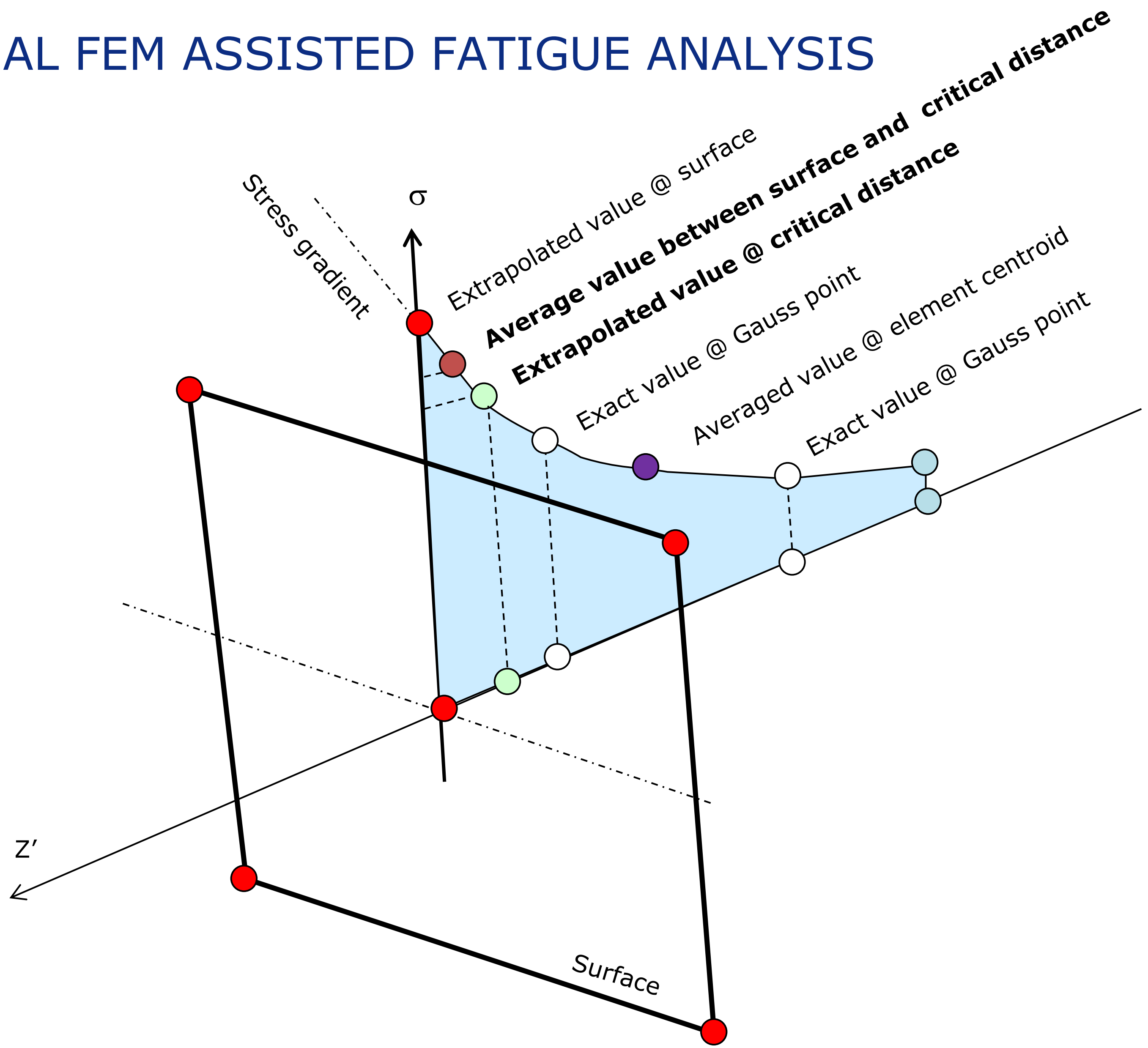
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS



MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS



MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

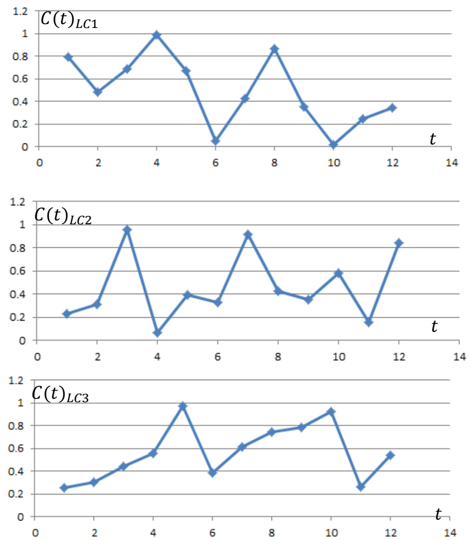


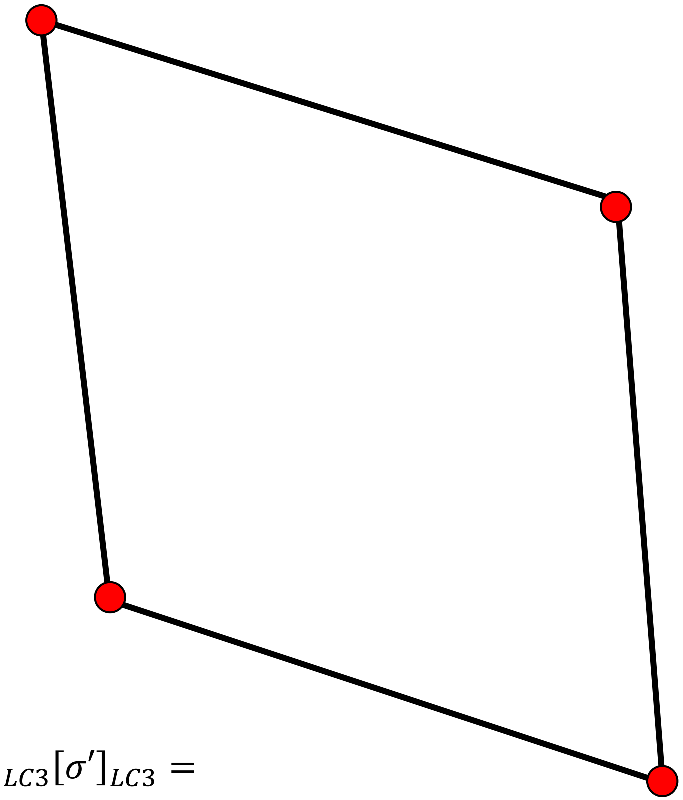
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

$$[\sigma']_{LC1} = \begin{bmatrix} \sigma'_{xx} & \sigma'_{xy} \\ \sigma'_{xy} & \sigma'_{yy} \end{bmatrix}_{LC1}$$

$$[\sigma']_{LC2} = \begin{bmatrix} \sigma'_{xx} & \sigma'_{xy} \\ \sigma'_{xy} & \sigma'_{yy} \end{bmatrix}_{LC2}$$

$$[\sigma']_{LC3} = \begin{bmatrix} \sigma'_{xx} & \sigma'_{xy} \\ \sigma'_{xy} & \sigma'_{yy} \end{bmatrix}_{LC3}$$





$$[\sigma'(t)]_{tot} = \begin{bmatrix} \sigma'_{xx}(t) & \sigma'_{xy}(t) \\ \sigma'_{xy}(t) & \sigma'_{yy}(t) \end{bmatrix} = C(t)_{LC1}[\sigma']_{LC1} + C(t)_{LC2}[\sigma']_{LC2} + C(t)_{LC3}[\sigma']_{LC3} =$$

$$= C(t)_{LC1} \begin{bmatrix} \sigma'_{xx} & \sigma'_{xy} \\ \sigma'_{xy} & \sigma'_{yy} \end{bmatrix}_{LC1} + C(t)_{LC2} \begin{bmatrix} \sigma'_{xx} & \sigma'_{xy} \\ \sigma'_{xy} & \sigma'_{yy} \end{bmatrix}_{LC2} + C(t)_{LC3} \begin{bmatrix} \sigma'_{xx} & \sigma'_{xy} \\ \sigma'_{xy} & \sigma'_{yy} \end{bmatrix}_{LC3} = \sum_i C(t)_{LCi}[\sigma']_{LCi}$$

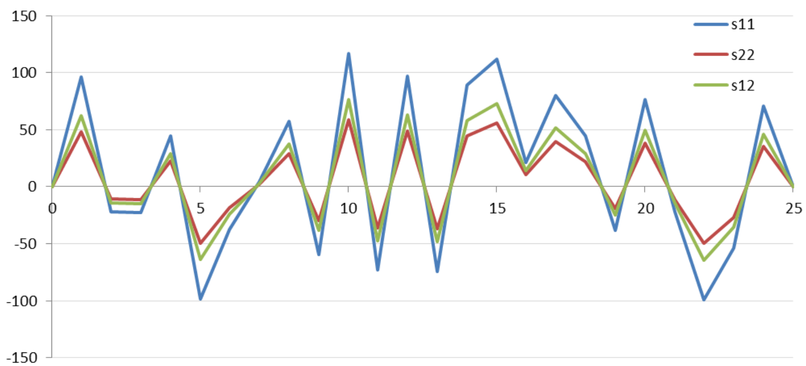
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

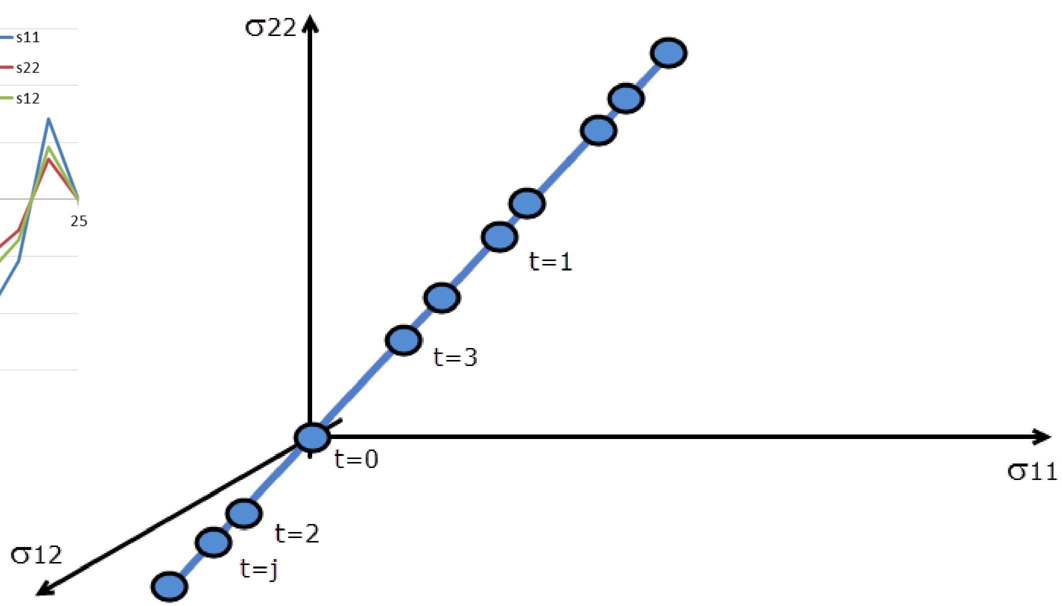
- Models and methods described in Part 1 relate to UNIAXIAL conditions
  - One stress or one strain (with its own time history),
  - The fatigue parameters is built with one stress or one strain

- Dealing with Multiaxial Stress Tensors, the Fatigue analysis problem gets significantly more complex.
- Depending on the nature of the applied loads, the Multiaxial problems are divided into two categories:
  - Multiaxial – Proportional Loadings
  - Multiaxial – Non-Proportional Loadings

MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

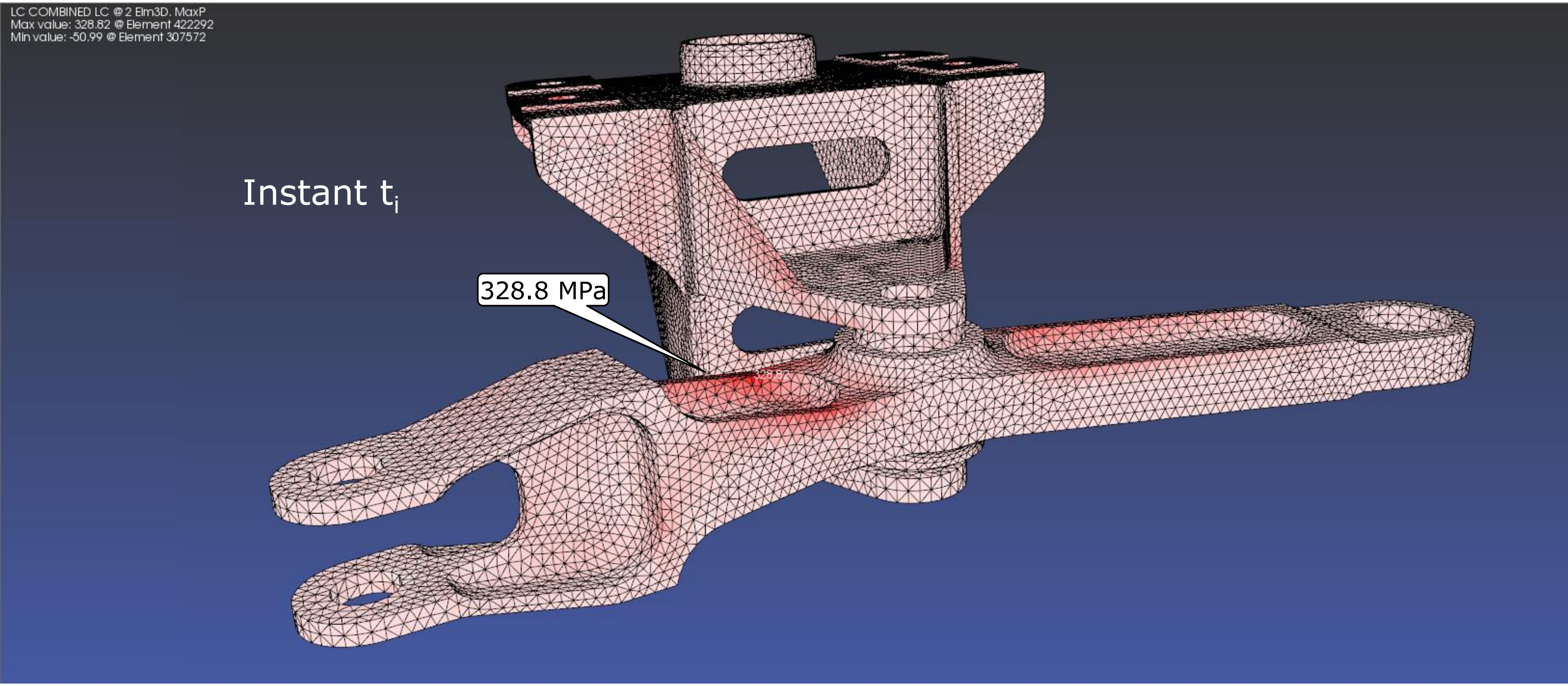
- Multiaxial – Proportional Loadings.
  - This situation typically occurs when the structure is subjected to a single load, whose magnitude changes over time, or when the structure is subjected to a set of loads which change all in phase over time





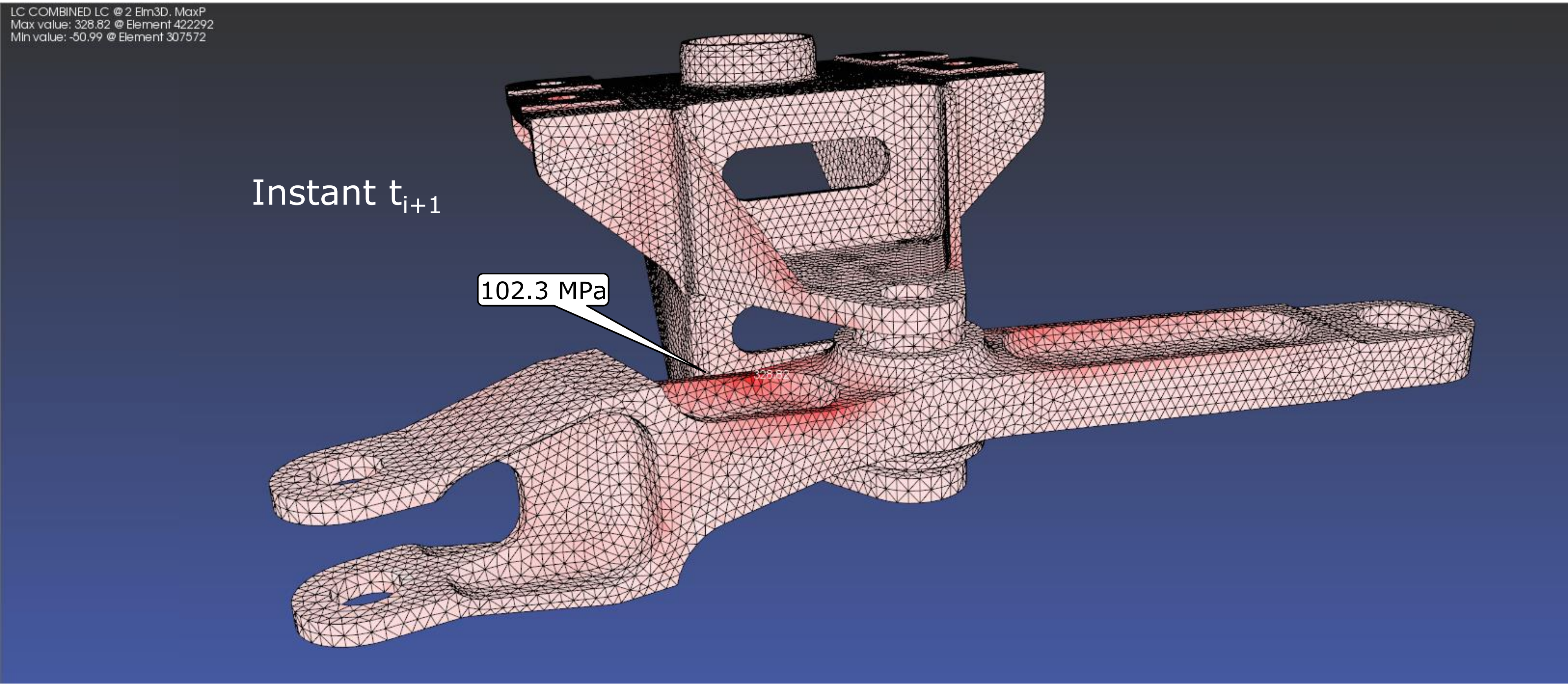
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Multiaxial Proportional Loading Conditions (software LIFING)



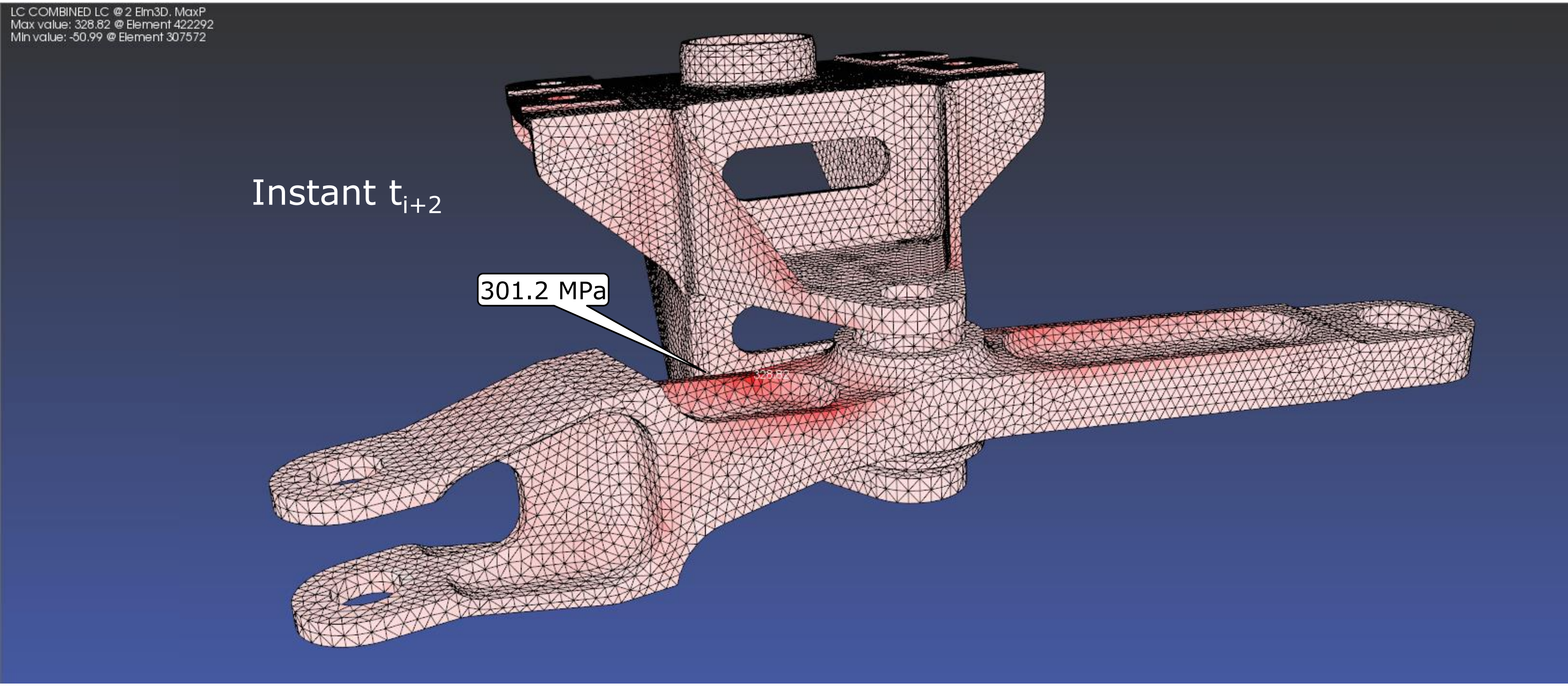
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Multiaxial Proportional Loading Conditions (software LIFING)



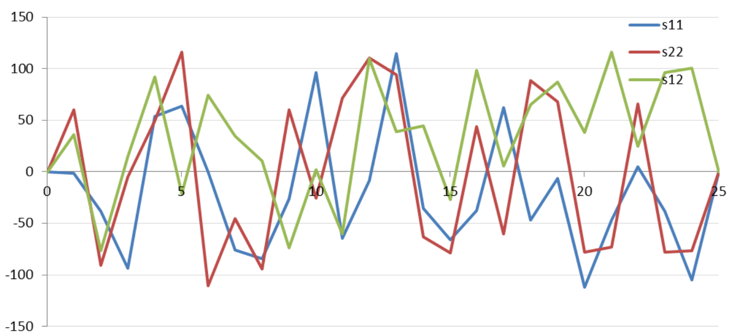
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

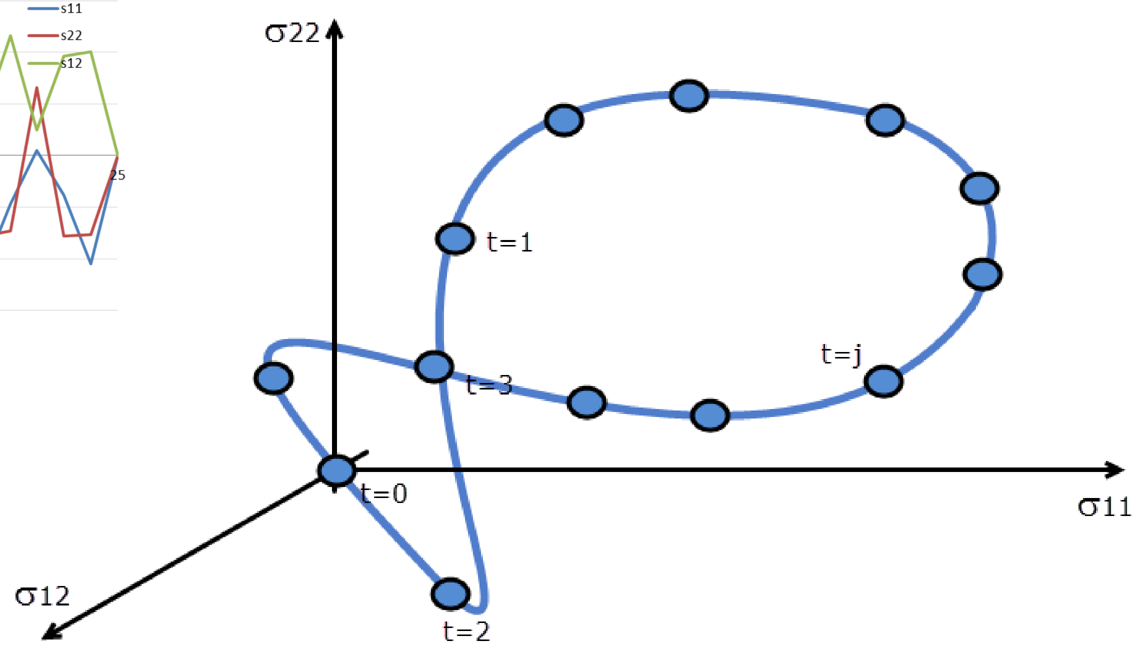
- Multiaxial Proportional Loading Conditions (software LIFING)



MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

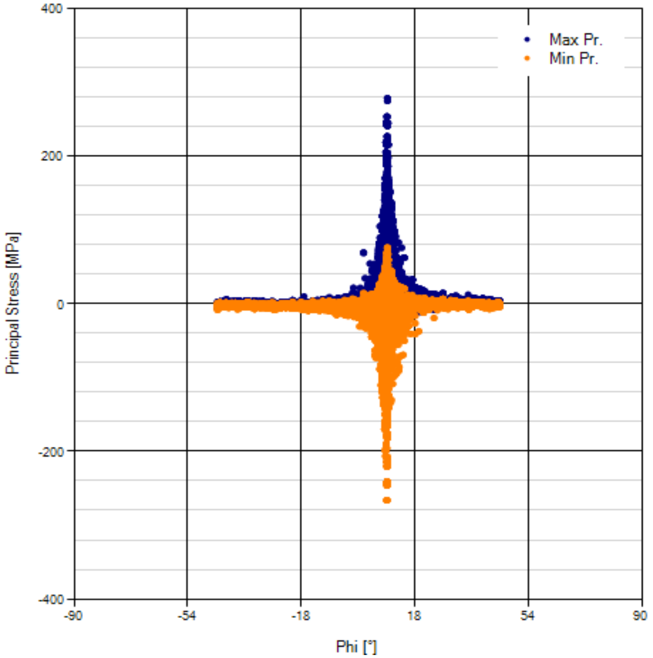
- Multiaxial – Non-Proportional Loadings.
  - This situation is the general one, when the structure is subjected to multiple loads which vary in time not in phase.

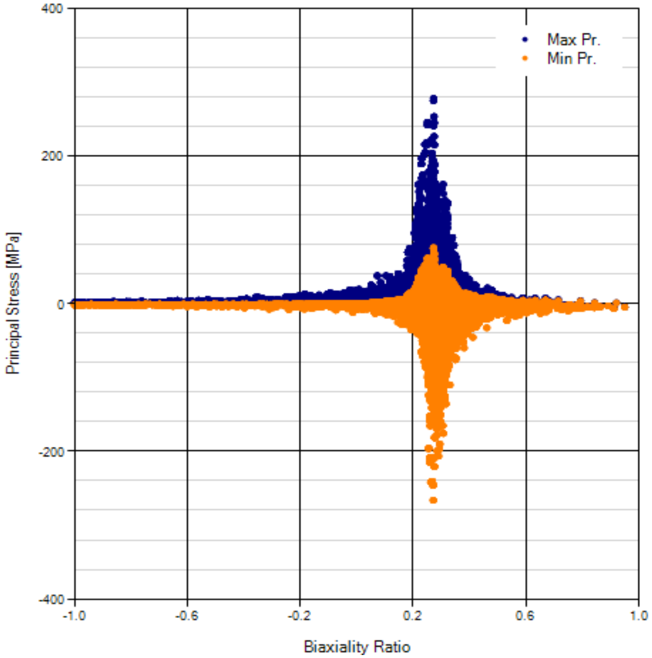




MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

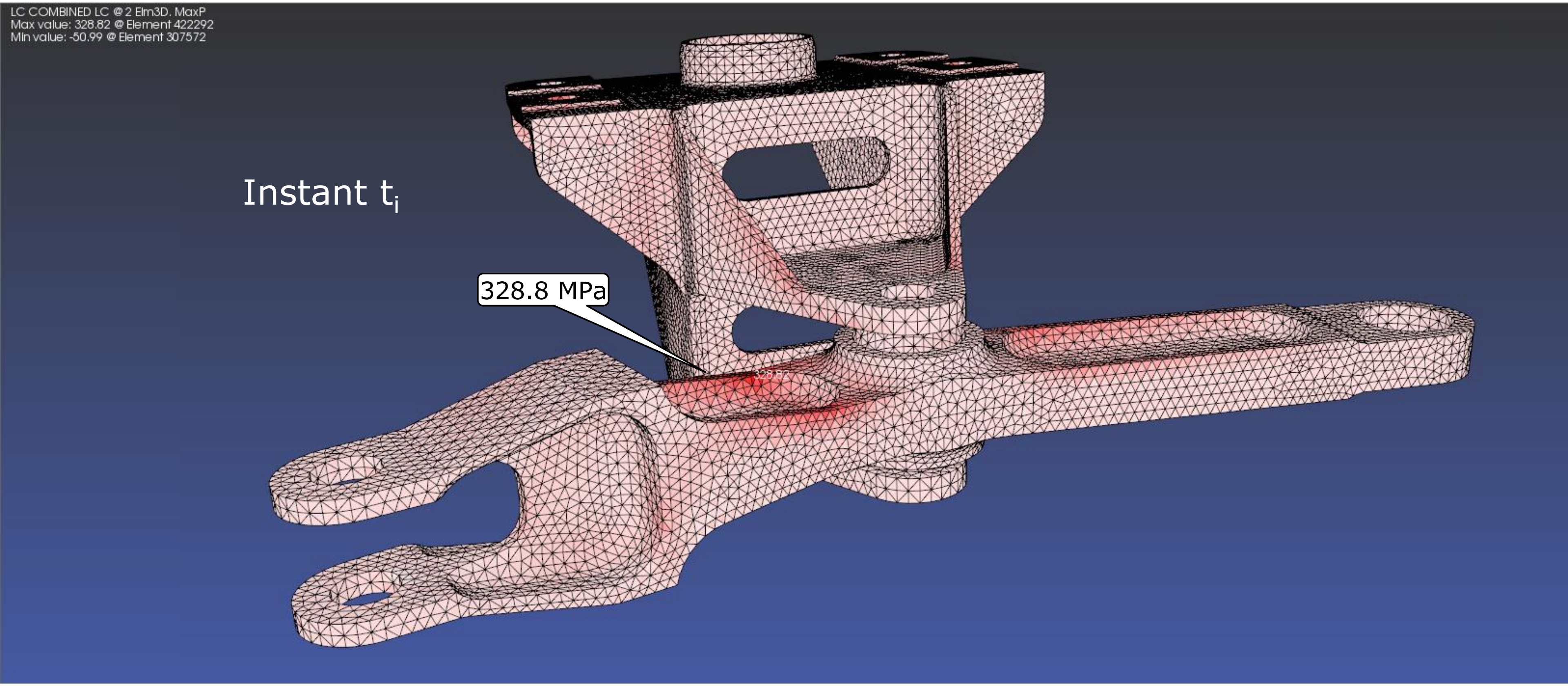
- Multiaxial – Non-Proportional Loadings.
  - If stress components are plotted in a chart, the points do not lay on a not straight line.
  - Similarly, stress principal directions and bi-axiality ratios change over time.





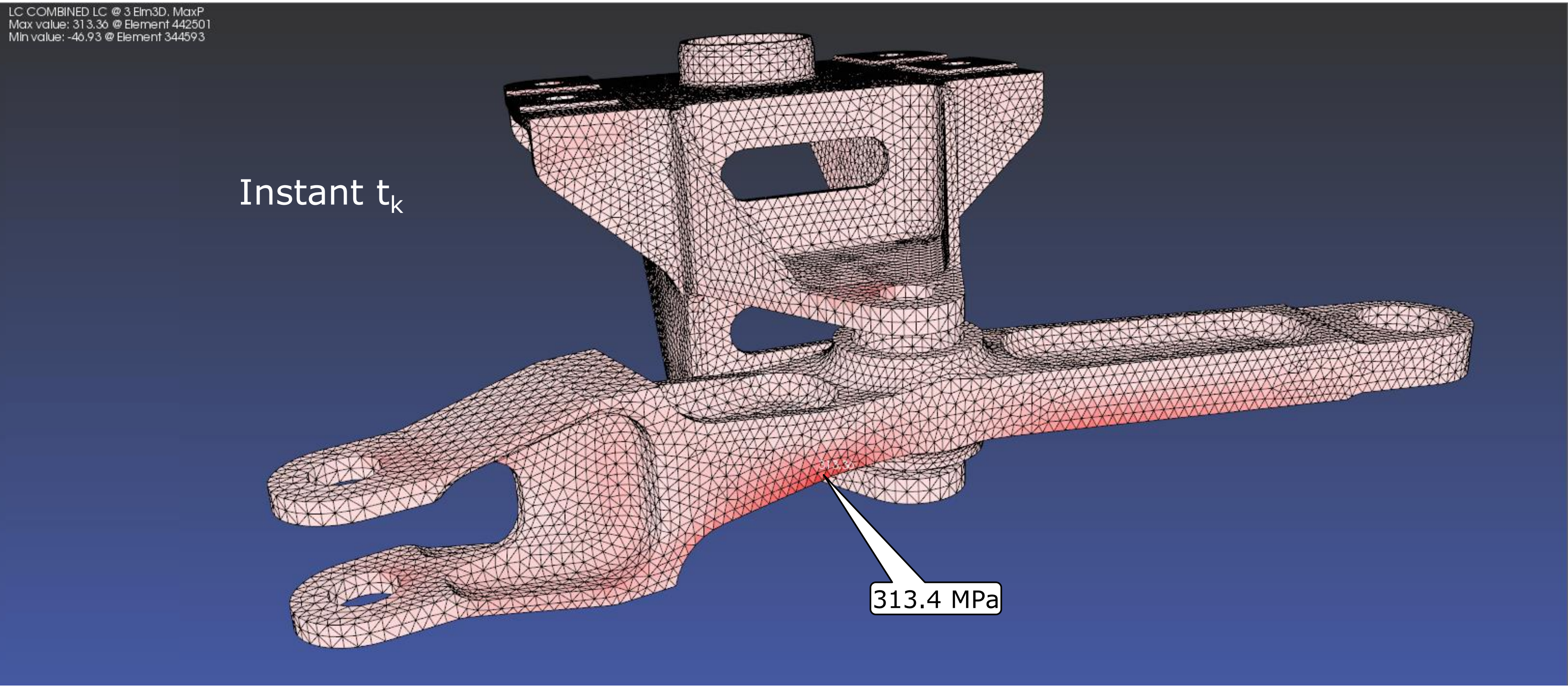
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Multiaxial Non-Proportional Loading Conditions (software LIFING)



MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Multiaxial Non-Proportional Loading Conditions (software LIFING)



MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Biaxiality Ratio is usually defined as the ratio between the Minimum and Maximum (in magnitude) Principal Stress.

$$[\sigma] = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \qquad \lambda = \frac{\sigma_2}{\sigma_1}$$

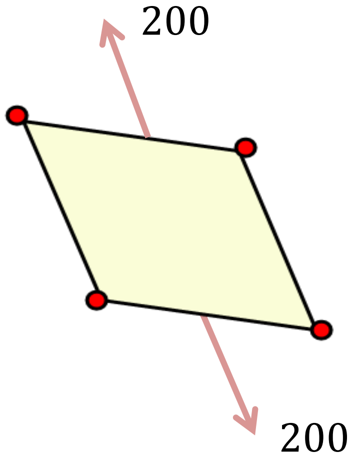
- By definition, the bi-axiality ratio λ spans between -1 and 1.
- In case of uniaxial stress tensor, the bi-axiality λ ratio is zero.
- What could be the implication of $\lambda \neq 0$ in a calculation which neglects the presence of a second stress? (*)

(*) This is the case, for example, when the analysis is carried out just looking at the Maximum Principal stress time history

MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

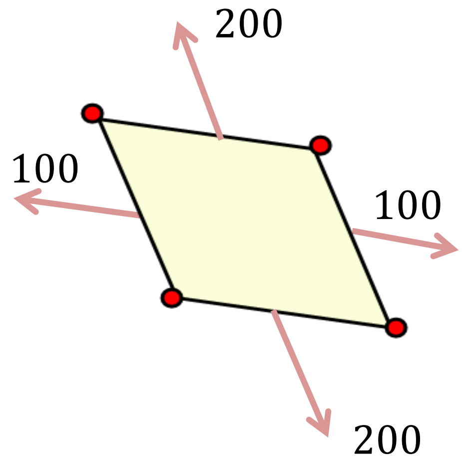
- What would happen if we run a fatigue analysis at three different notches loaded with the same time history of loads (Multiaxial Proportional Loading), where the following three reference stress tensors are given?

Case 1



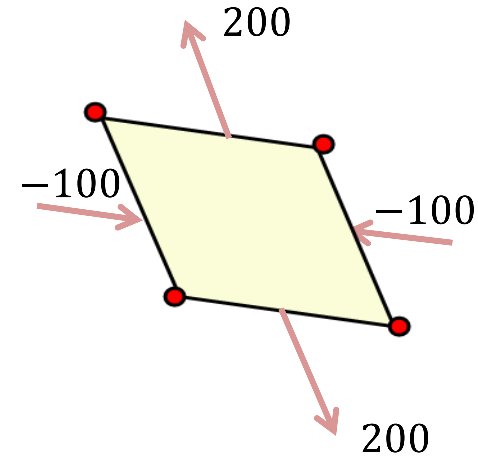
$$\begin{bmatrix} 200 & 0 \\ 0 & 0 \end{bmatrix}$$

Case 2



$$\begin{bmatrix} 200 & 0 \\ 0 & 100 \end{bmatrix}$$

Case 3

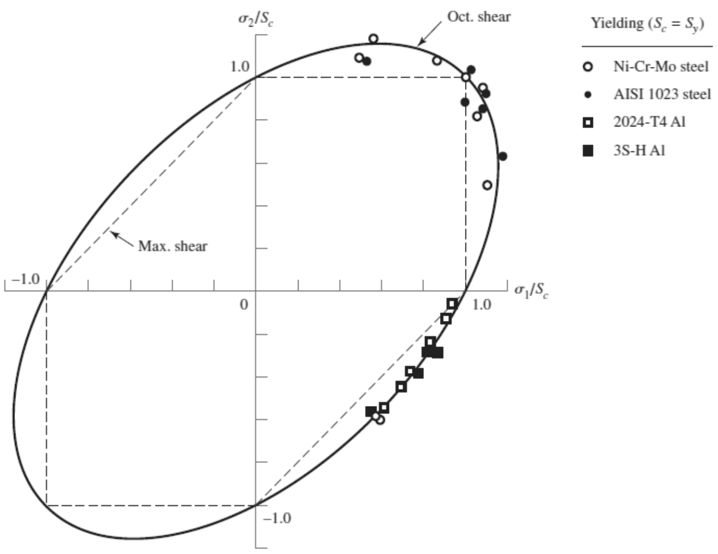


$$\begin{bmatrix} 200 & 0 \\ 0 & -100 \end{bmatrix}$$

MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

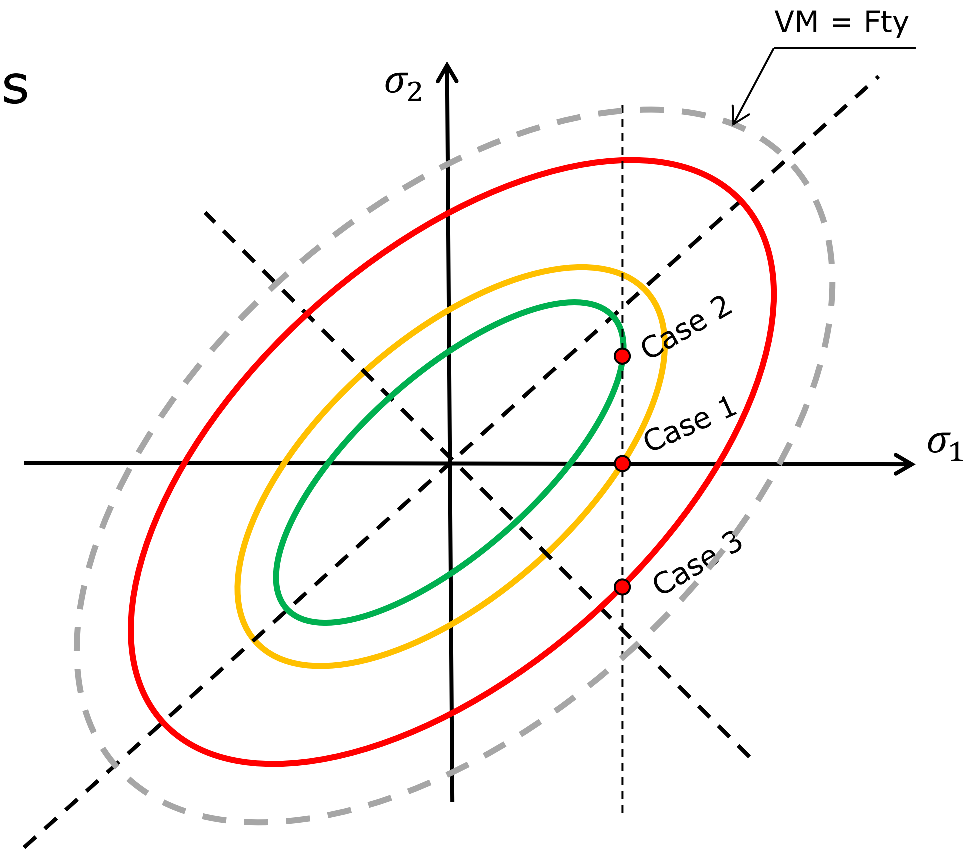
- The fatigue analysis performed on the basis of the Max Principal Stress would deliver the same result for all the three cases. However if we calculate the Von Mises stresses we have:

- Case 1. $\mathrm{VM} = \sqrt{{\sigma_1}^2 + {\sigma_2}^2 - \sigma_1 \cdot \sigma_2} = 200\mathrm{MPa}$
- Case 2. $\mathrm{VM} = \sqrt{{\sigma_1}^2 + {\sigma_2}^2 - \sigma_1 \cdot \sigma_2} = 173.2\mathrm{MPa}$
- Case 3. $\mathrm{VM} = \sqrt{{\sigma_1}^2 + {\sigma_2}^2 - \sigma_1 \cdot \sigma_2} = 264.6\mathrm{MPa}$



MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Yield surfaces



Case 1: 200 MPa

Case 2: 173.2 MPa

Case 3: 264.6 MPa

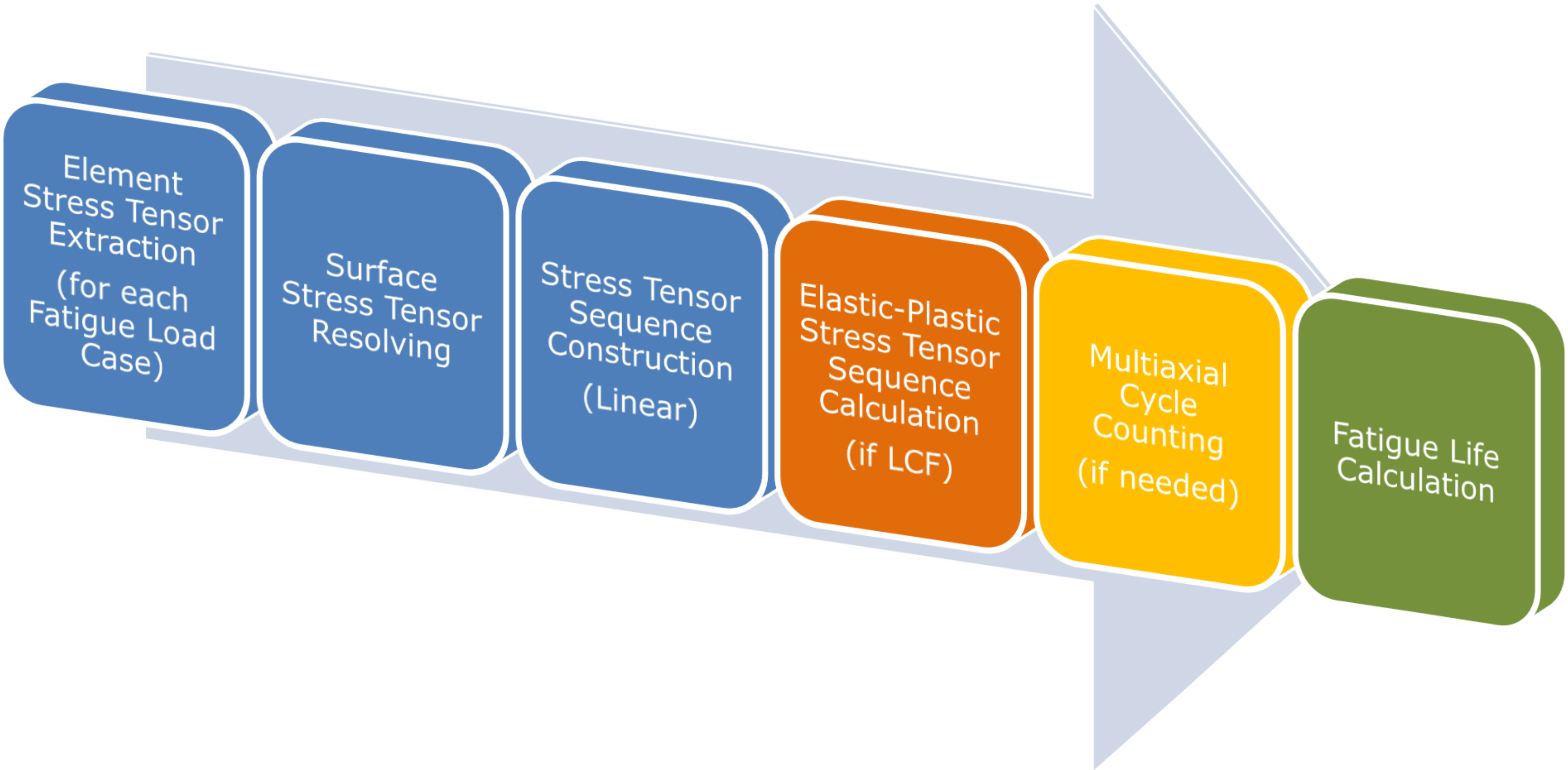
- Can the three cases be equally damaging?

MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- The fatigue (crack initiation) analysis performed on the basis of the Max Principal Stress, inherits two fundamental errors:

  - (1) Assume the problem is Multiaxial Proportional Loading.
    - The impact of biaxiality ratio is ignored, meaning that:
      - CONSERVATIVE ERROR if $\lambda>0$
      - UNCONSERVATIVE ERROR if $\lambda<0$
  - (2) Assume the problem is Multiaxial Non-Proportional Loading.
    - The Maximum Principal Plane ROTATES over the time (and biaxiality ratio changes as well), meaning that:
      - At each instant in the time history the structure wants to crack at different planes, whereas the Maximum Principal is an 'invariant' (i.e. plane insensitive)

MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS



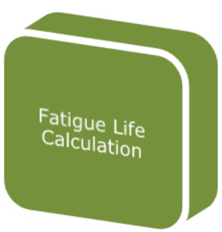
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- The Non-Proportionality significantly increases the problem complexity, because of the following issues:

  - Solving Cyclic Plasticity (in case of LCF) with multiple stress components is way more complex (many methods are available), therefore the calculation of elastic-plastic stress-strains out of elastic FEM calculated stress-strains is a very complex issue.

    (Simple approaches are available in case of Proportional Loadings)

  - If we were able to calculate elastic-plastic stress-strains, defining cycles within a Stress Tensor time history where components change over the time not in phase is complex (Wang-Brown method is proposed in literature, however some analysis methods do not require special sequence counting).

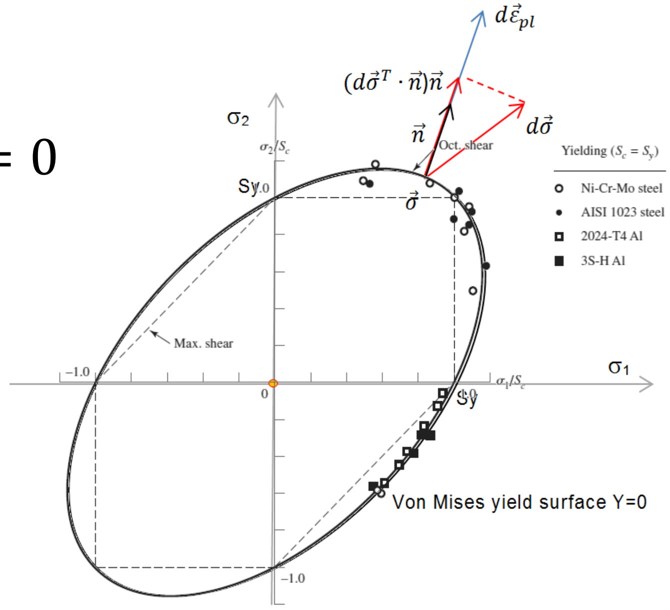


  - What is the best fatigue parameter, combination of stress components, if multiple stress components vary over the time?

MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Cyclic Plasticity Calculation
  - Yield function

$$Y = \sigma_1{}^2 + \sigma_2{}^2 - \sigma_1 \cdot \sigma_2 - Sy^2 = 0$$



  - Plastic Flow ("normality") Rule

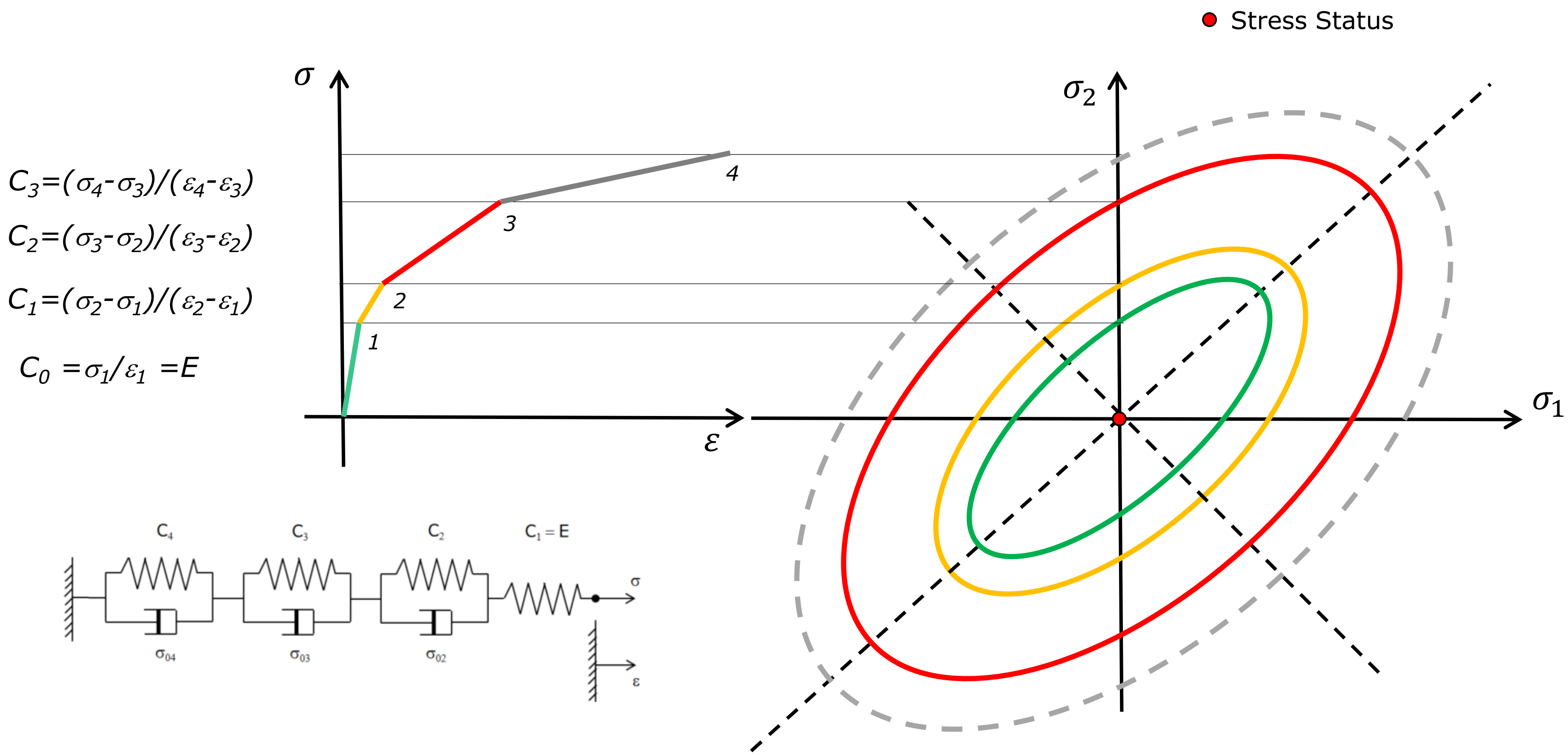
$$d\vec{\varepsilon}_{pl} = \frac{1}{C}(d\vec{\sigma}^T \cdot \vec{n}) \cdot \vec{n}$$

C = Plastic modulus

  - Hardening Rule
    - Many models have been developed, for example the *Mróz-Garud* Multi-Surface (hardening) Model

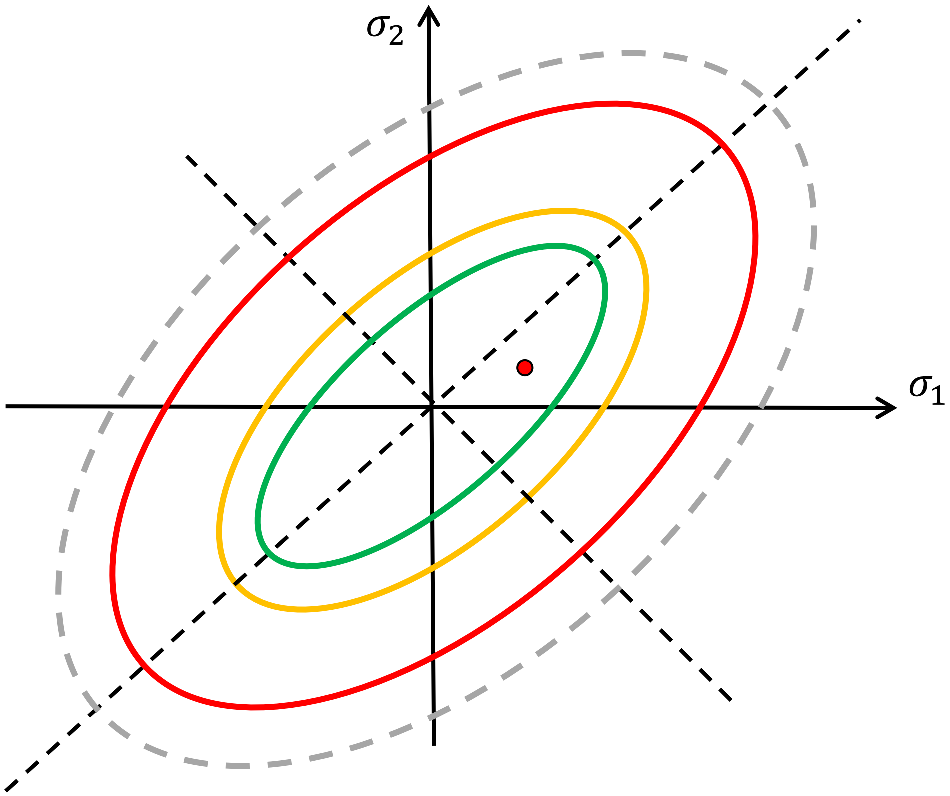
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Instant 0 $(\vec{\sigma} = \vec{0})$



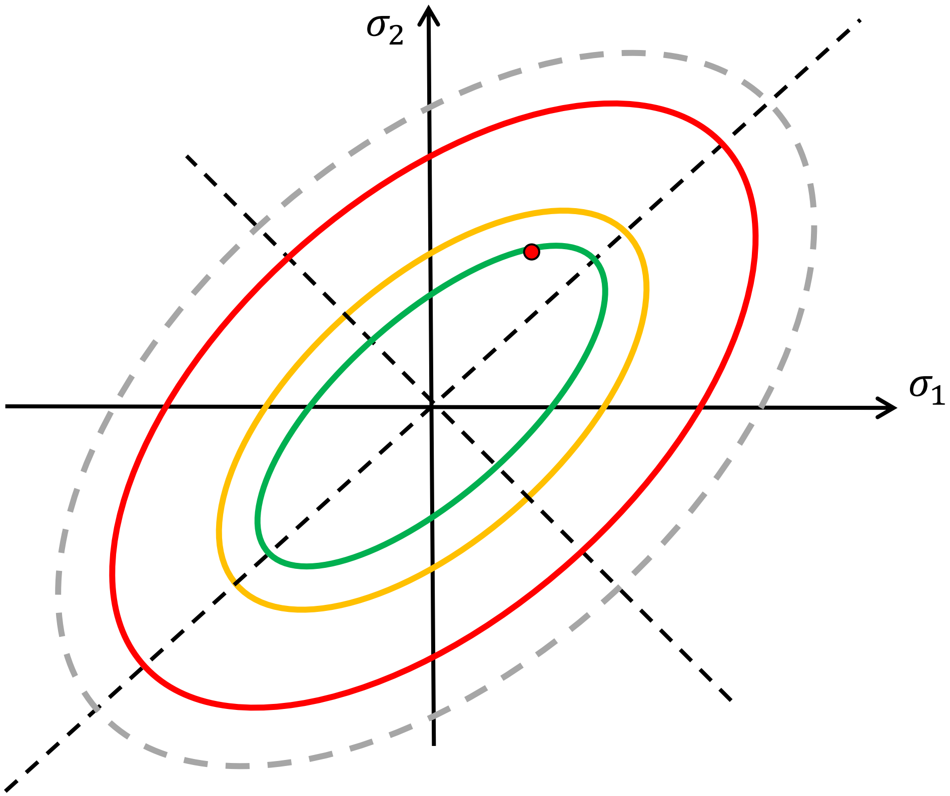
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Instant 0
- Instant 1



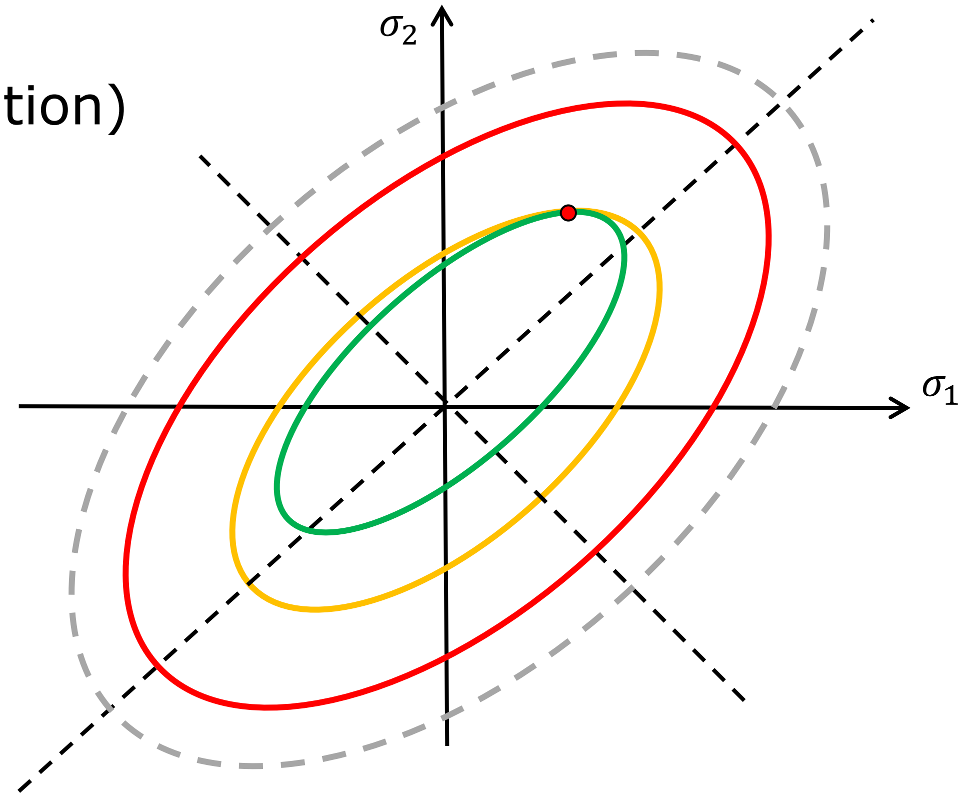
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Instant 0
- Instant 1
- Instant 2 (yielding)



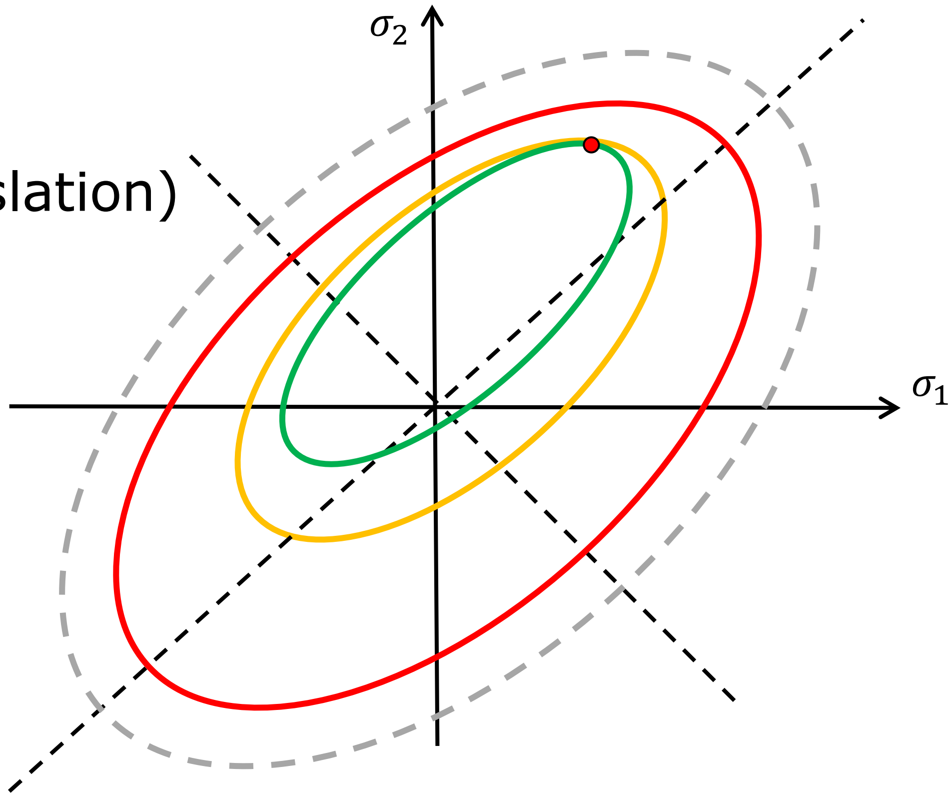
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Instant 0
- Instant 1
- Instant 2
- Instant 3 (yielding surf. translation)



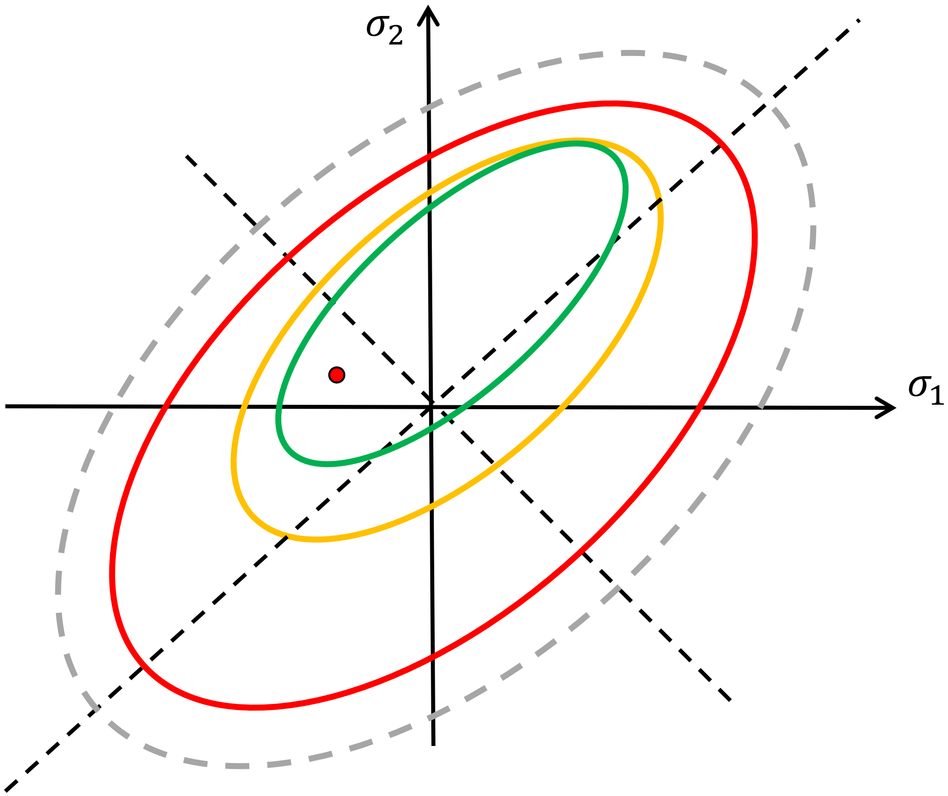
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Instant 0
- Instant 1
- Instant 2
- Instant 3
- Instant 4 (hardening surf. translation)



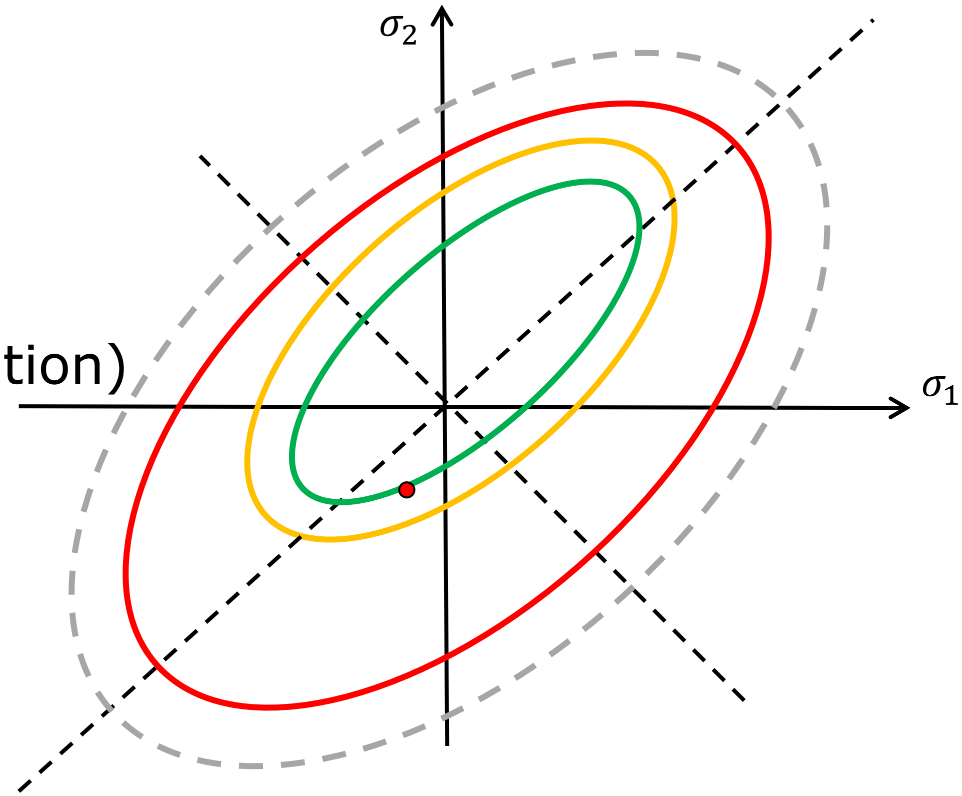
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Instant 0
- Instant 1
- Instant 2
- Instant 3
- Instant 4
- Instant 5



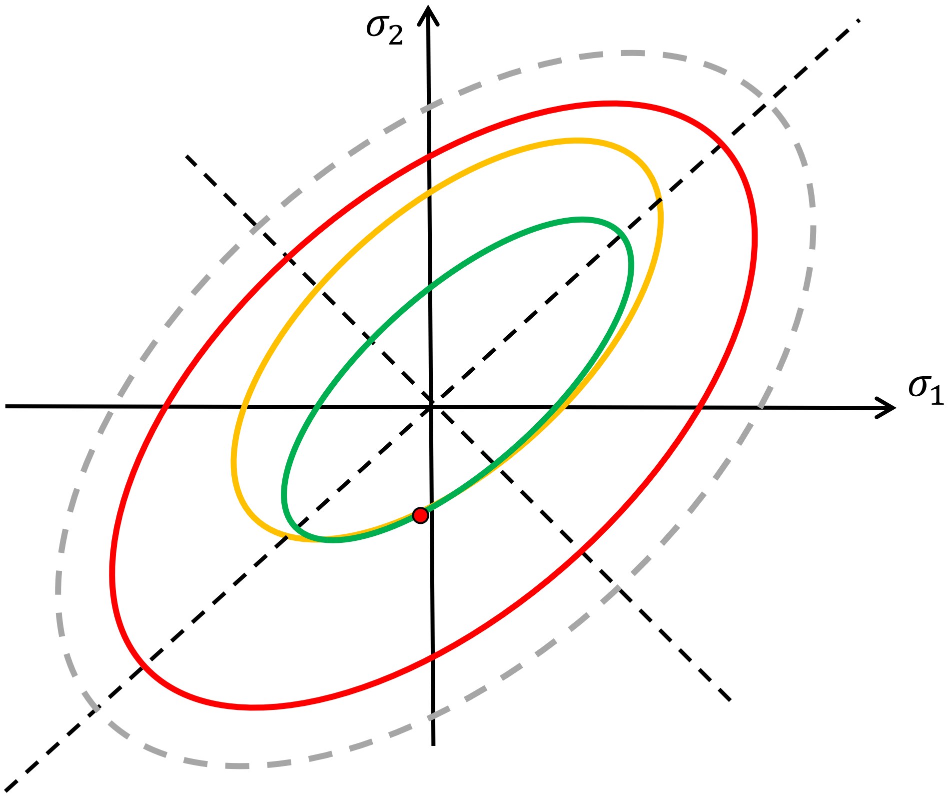
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Instant 0
- Instant 1
- Instant 2
- Instant 3
- Instant 4
- Instant 5
- Instant 6 (yielding surf. translation)



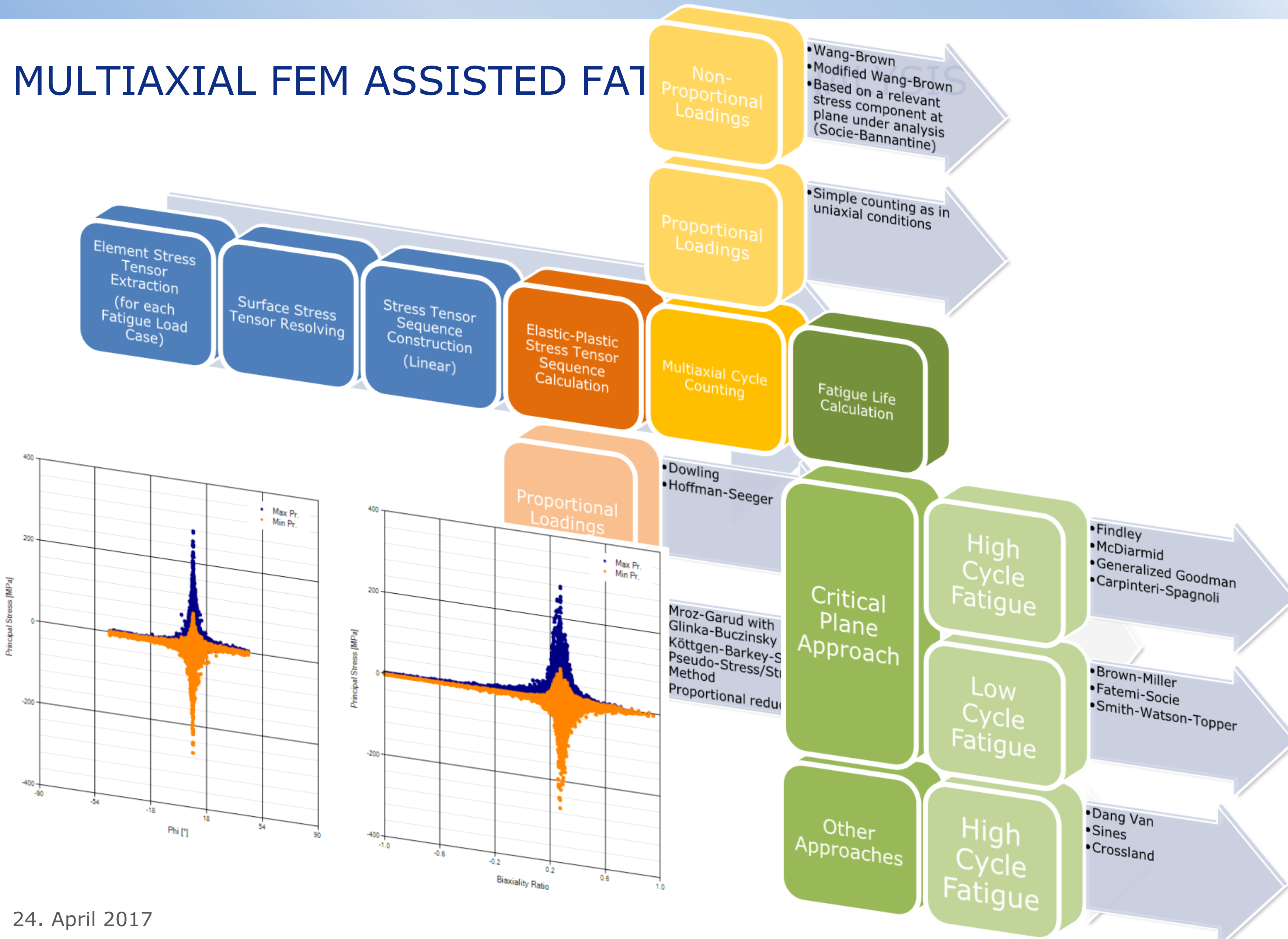
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Instant 0
- Instant 1
- Instant 2
- Instant 3
- Instant 4
- Instant 5
- Instant 6
- Instant 7
- …



COMPUTATIONALLY DEMANDING
(INCREMENTAL APPROACH)

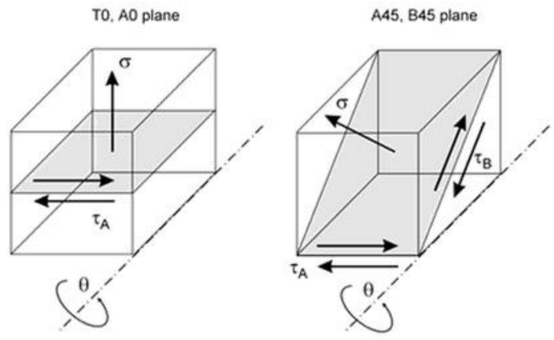
MULTIAXIAL FEM ASSISTED FAT



MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

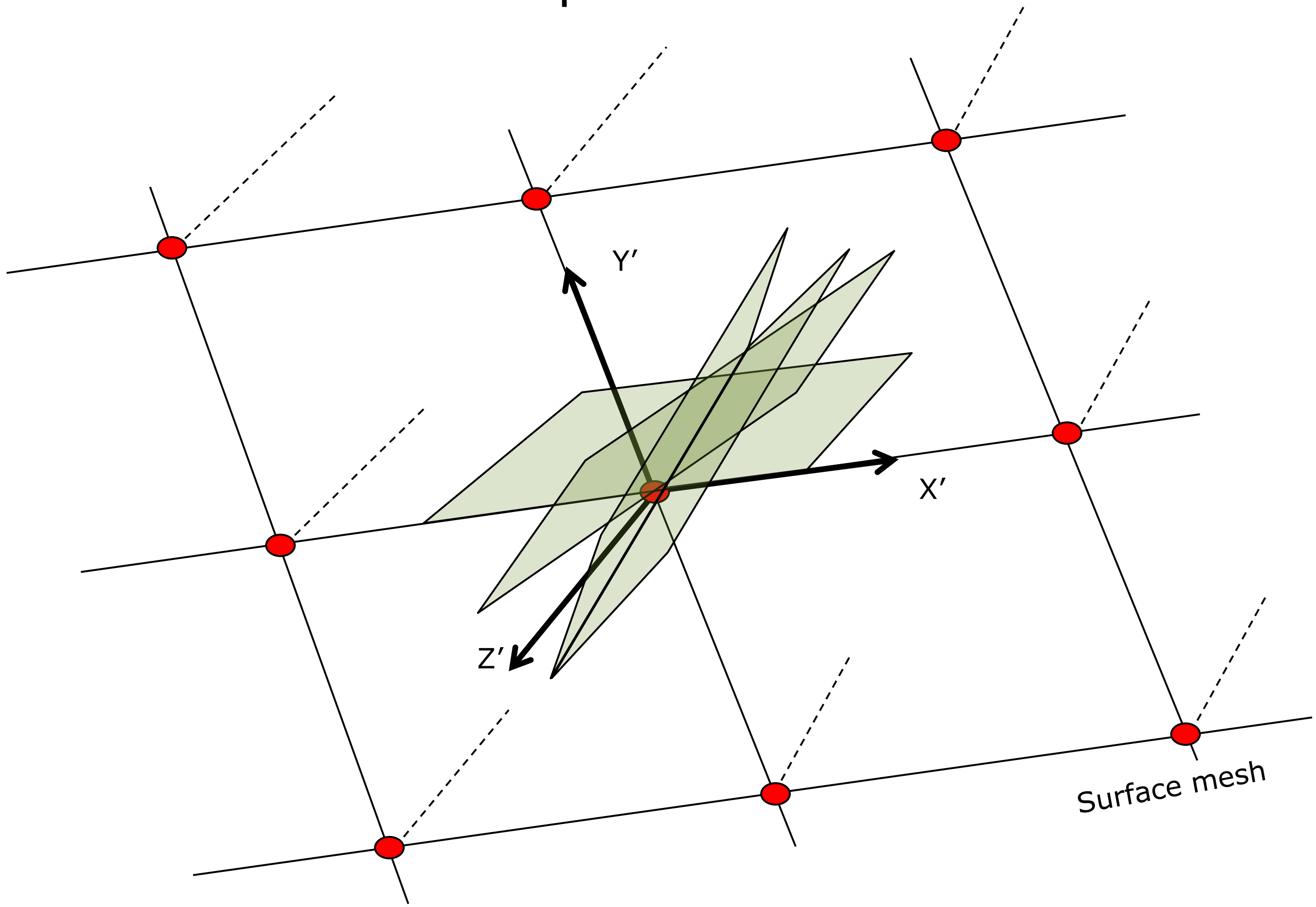
- Critical plane methods are very popular. They are based on analysis performed at many ‘candidate’ critical planes.

- Amongst all ‘candidate’ planes, the critical one is the plane which maximizes the defined fatigue parameter, e.g., in LCF:
  - Smith-Watson-Topper: $\sigma_{N,max} \cdot \frac{\Delta\varepsilon_N}{2}$
  - Brown-Miller: $\frac{\Delta\gamma_{max}}{2} + S\Delta\varepsilon_N$
  - Fatemi-Socie: $\frac{\Delta\gamma_{max}}{2}\left(1 + S\frac{\sigma_{N,max}}{S_y}\right)$



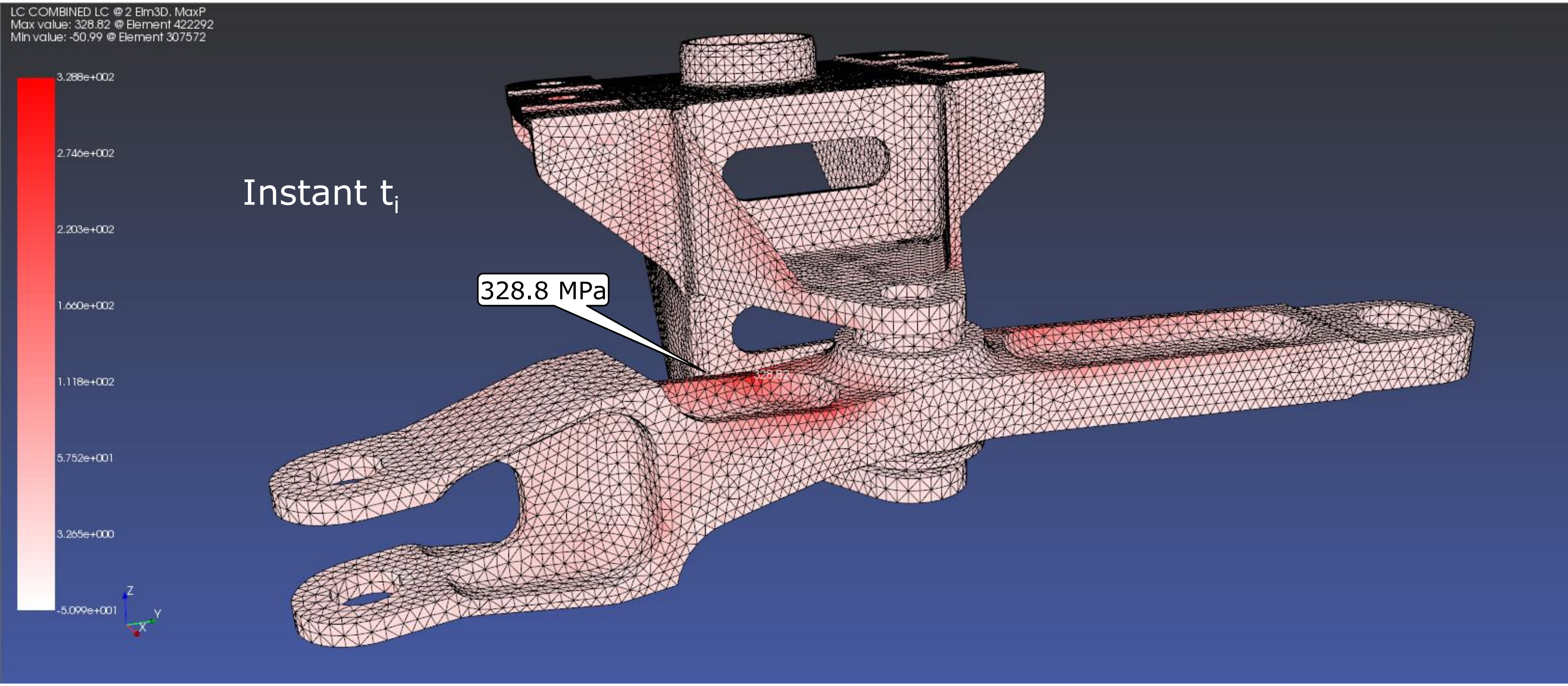
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Working on the surface of a mechanical component, two rotations define the critical plane.



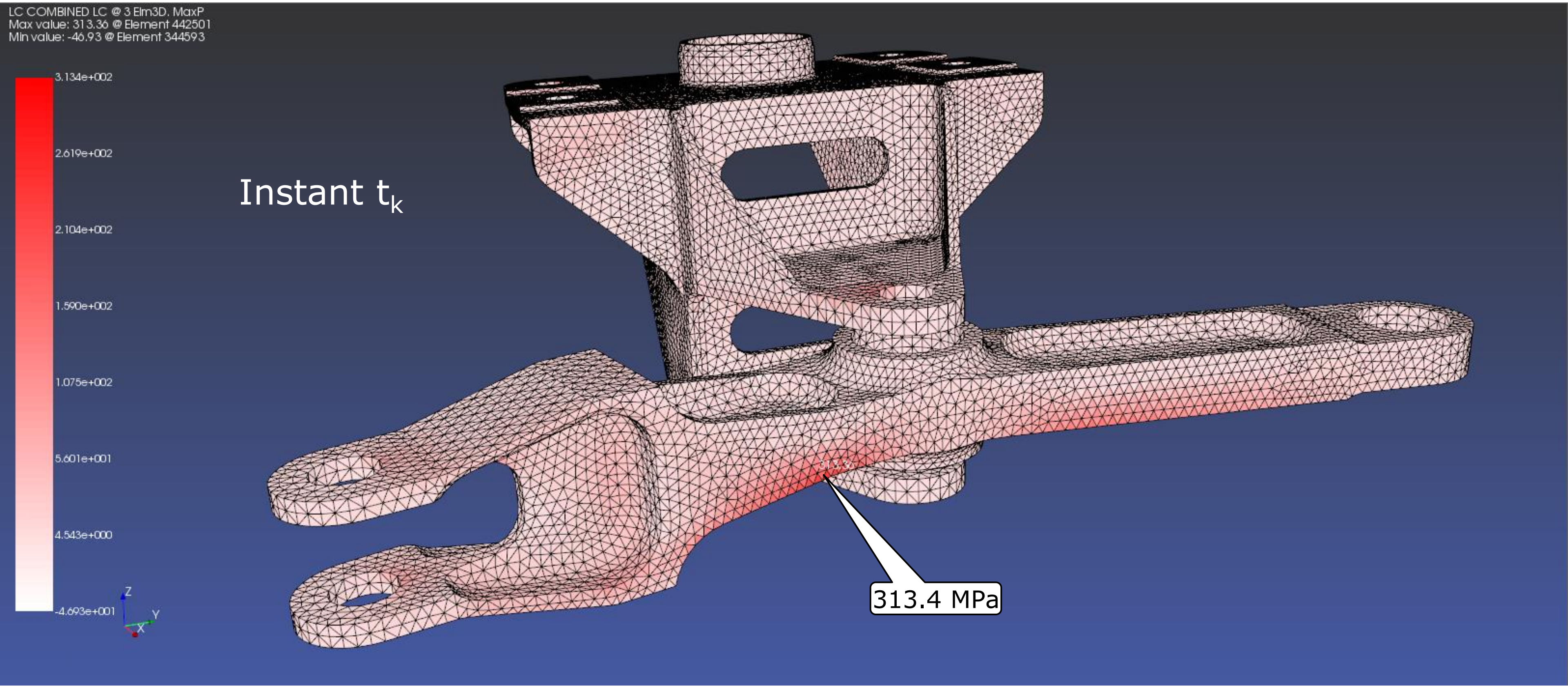
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Example with the software Lifing



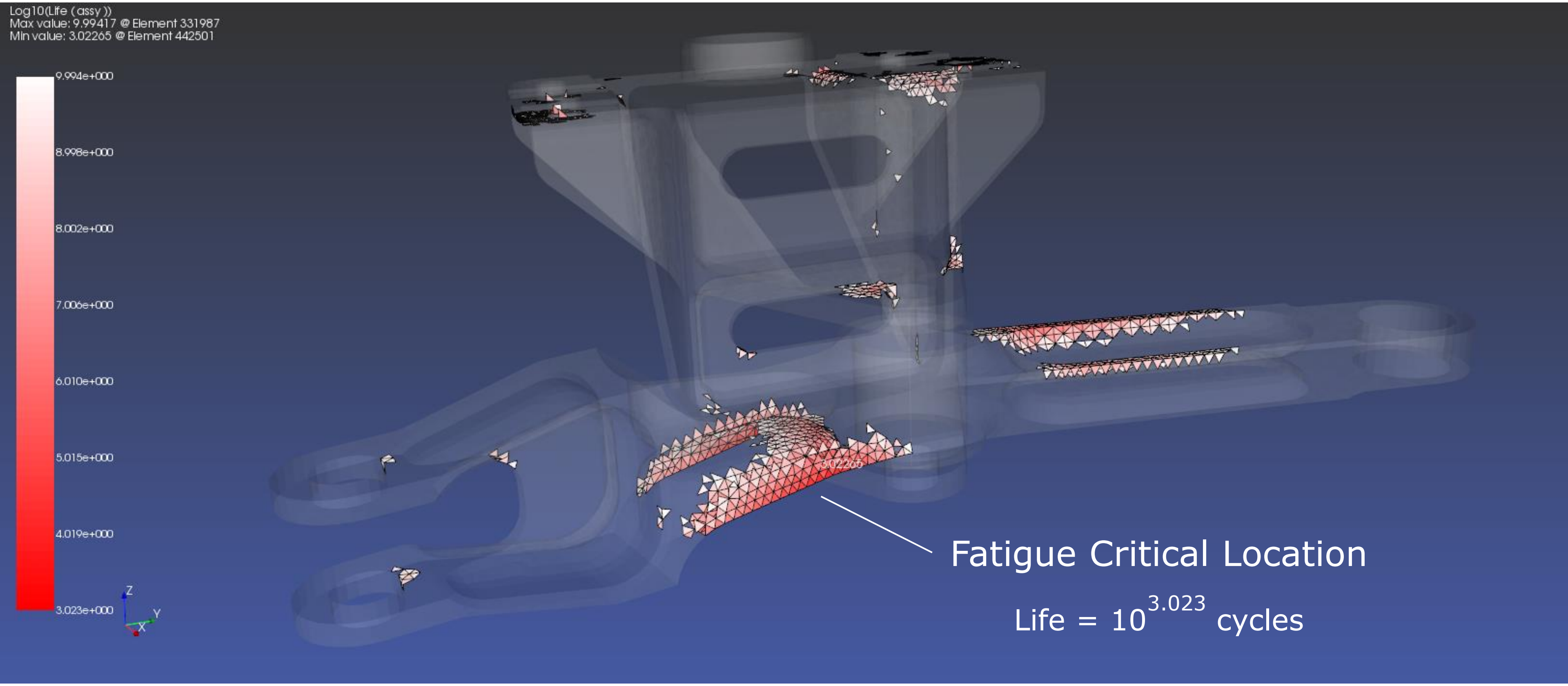
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Example with the software Lifing



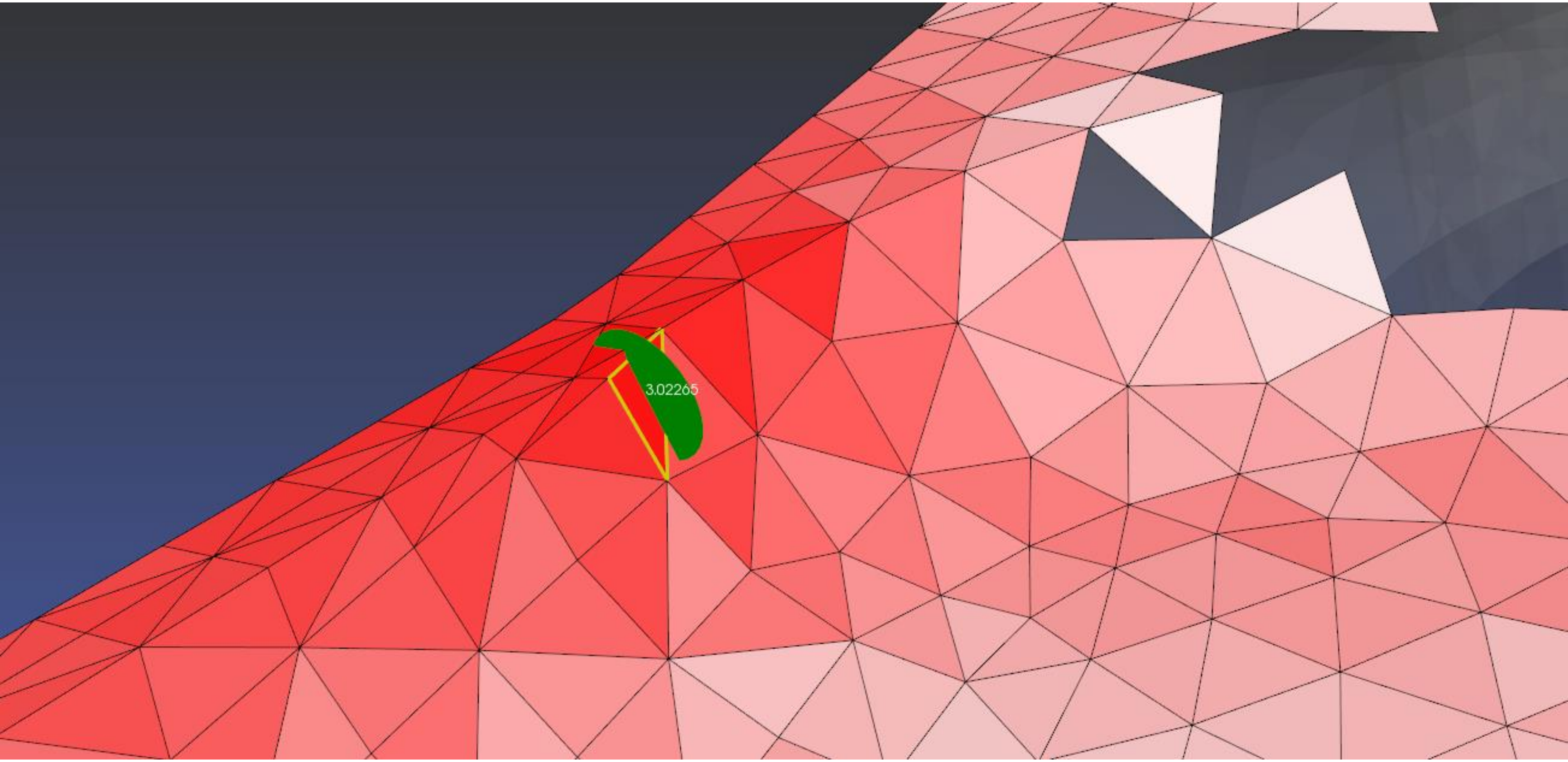
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Example with the software Lifing



MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Example with the software Lifing



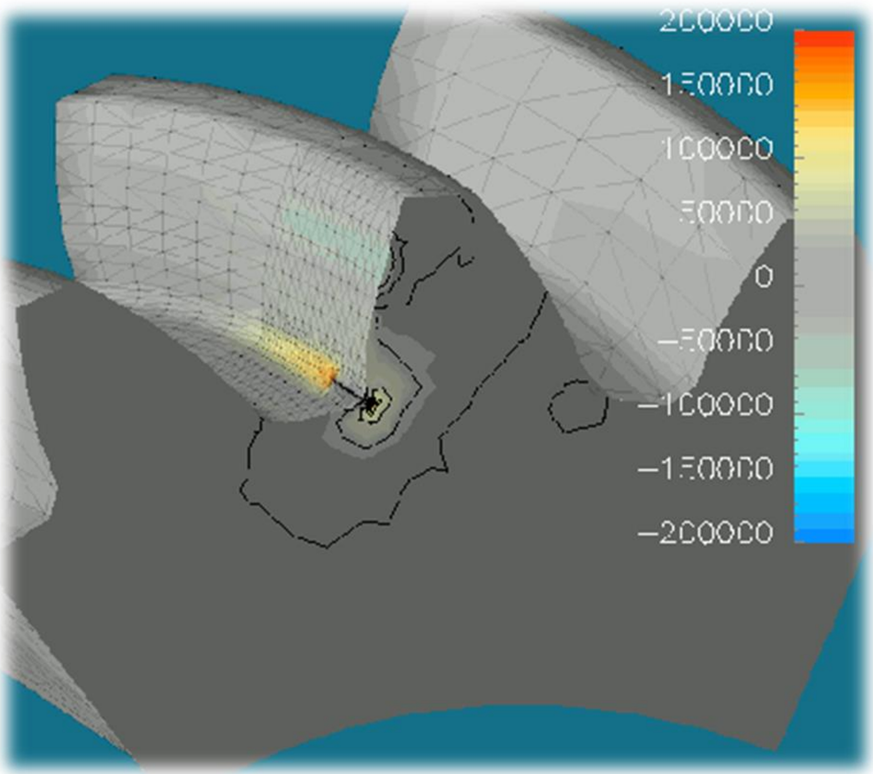
MULTIAXIAL FEM ASSISTED FATIGUE ANALYSIS

- Why the need of removing conservativism? ‘Fatigue analyses have always been done with conventional conservative approaches’...



- ...true, but the industry is going in the direction of ‘super-optimized’ structures. Converntional techniques are INSUFFICIENT! State-of-the-art analysis methods and tools are required

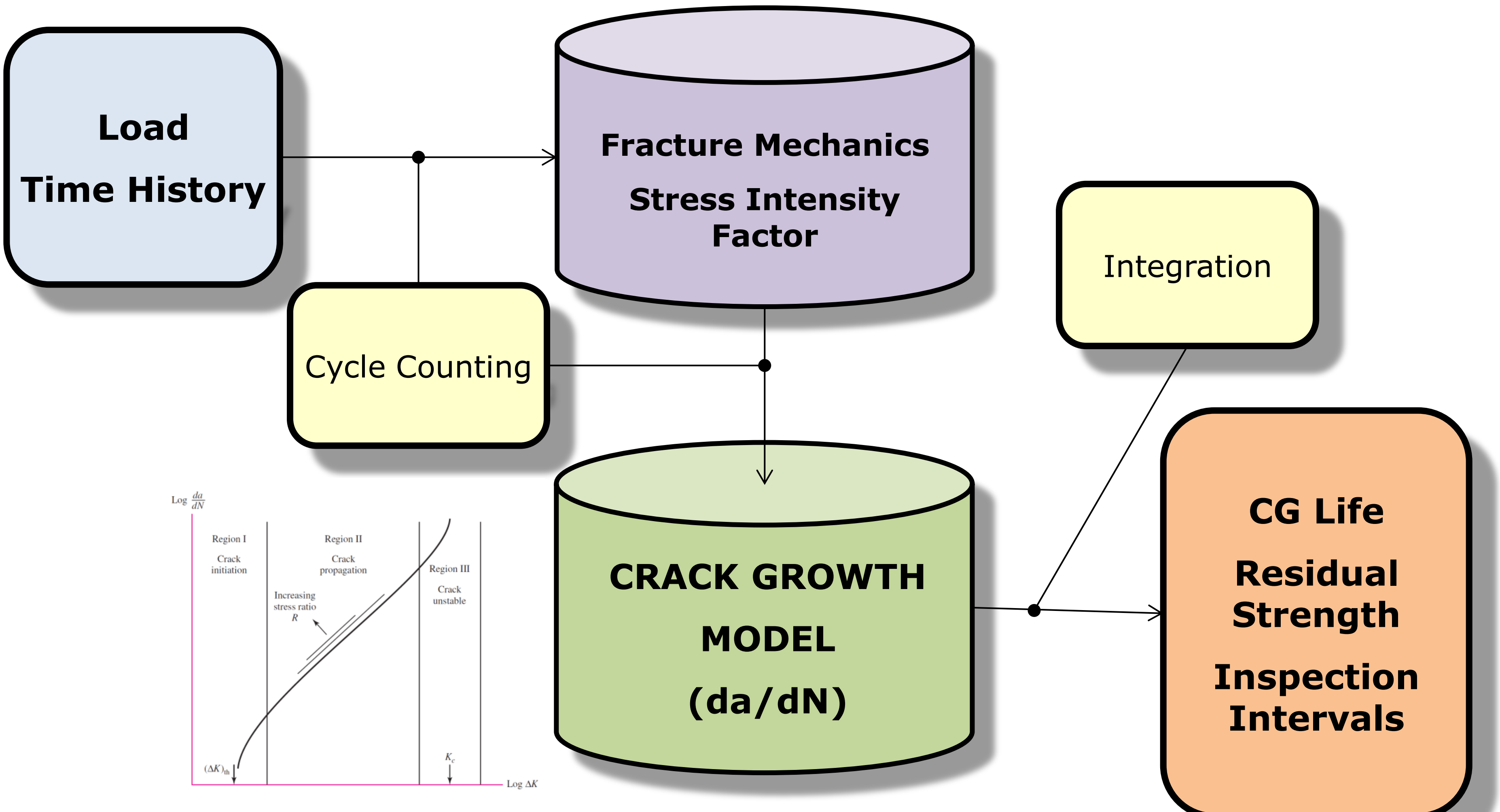
Part 3



DAMAGE TOLERANCE ANALYSIS

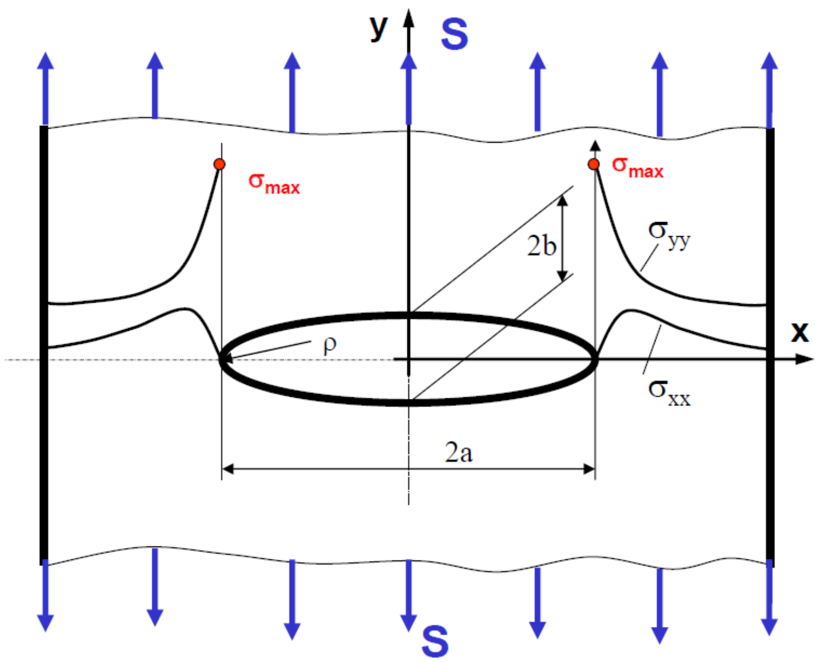
- The Damage Tolerance design criteria assume that cracks are always present in the structure (see F-111 accidents).
- The analyst shall demonstrate that crack growth is stable and…
- …that the structure has sufficient Residual Strength to survive missions up to the next maintenance.
- The calculation is also aimed to define inspection intervals.

DAMAGE TOLERANCE ANALYSIS



LINEAR ELASTIC FRACTURE MECHANICS

- Crack Tip Stress



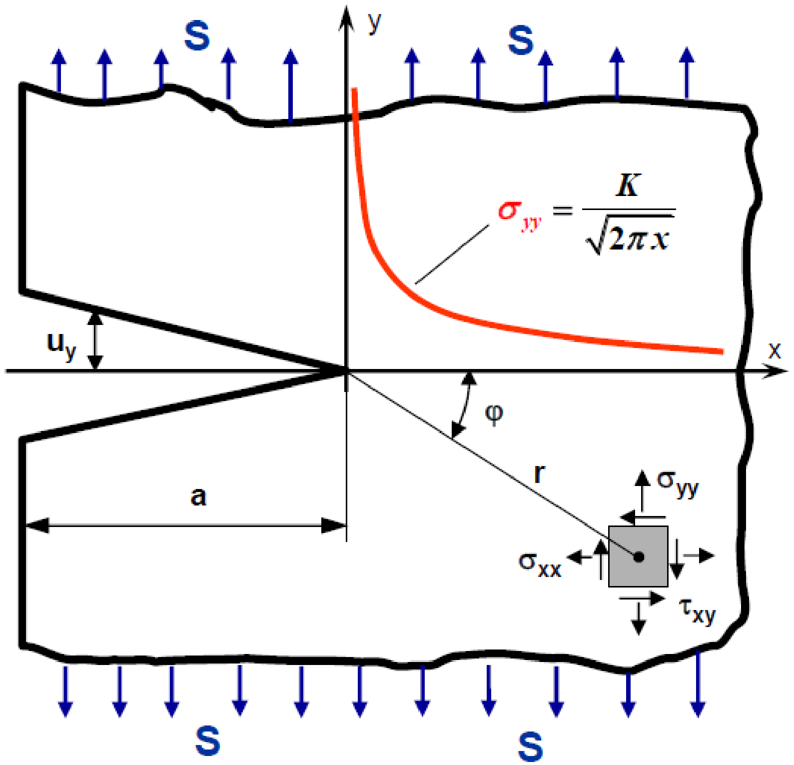
$$\sigma_{max} = K_t \cdot S$$

$$K_t = 1 + 2\sqrt{\frac{a}{\rho}}$$

- When ρ tends to 0 (the ellipse becomes a crack), the stress at the crack tip goes to infinity, which is a non-sense.
- In reality plasticity occurs; stress is finite.
- The linear elastic theories are no longer applicable at crack tips; the calculated stress is no longer a relevant quantity.

LINEAR ELASTIC FRACTURE MECHANICS

- Something else must be used to characterize the stress status at a crack tip: the Stress Intensity Factors.



- From Wastegaard and Irwin:

$$\sigma_{xx} = \frac{S\sqrt{\pi a}Y}{\sqrt{2\pi r}}\cos\left(\frac{\varphi}{2}\right)\left[1-\sin\left(\frac{\varphi}{2}\right)\sin\left(\frac{3\varphi}{2}\right)\right]+\psi_x(r,\varphi)$$

$$\sigma_{yy} = \frac{S\sqrt{\pi a}Y}{\sqrt{2\pi r}}\cos\left(\frac{\varphi}{2}\right)\left[1+\sin\left(\frac{\varphi}{2}\right)\sin\left(\frac{3\varphi}{2}\right)\right]+\psi_y(r,\varphi)$$

$$\tau_{xy} = \frac{S\sqrt{\pi a}Y}{\sqrt{2\pi r}}\cos\left(\frac{\varphi}{2}\right)\sin\left(\frac{\varphi}{2}\right)\cos\left(\frac{3\varphi}{2}\right)+\psi_{xy}(r,\varphi)$$

$$\sigma_{yy} = \sigma_{xx} = \frac{S\sqrt{\pi a}Y}{\sqrt{2\pi x}} \longrightarrow K_I = S\cdot\sqrt{\pi a}\cdot Y$$

for y = 0 and r<<a

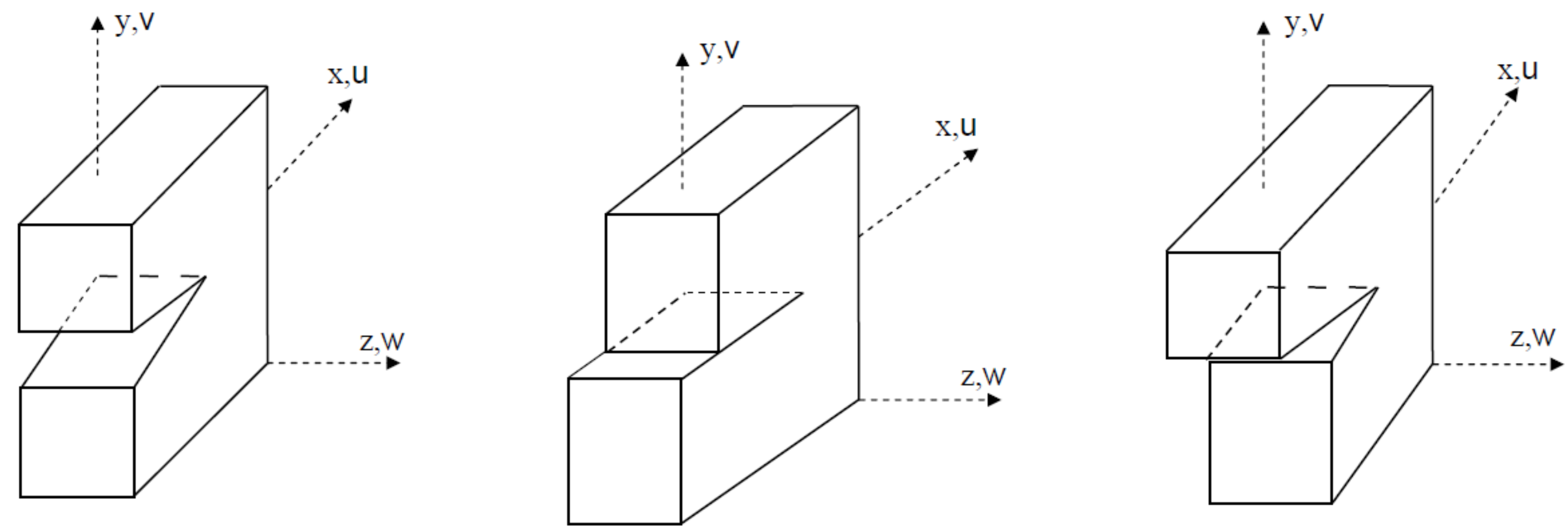
$MPa\sqrt{mm}$

$ksi\sqrt{inch}$

- Y is a geometrical factor, a is the crack size, S the remote stress.

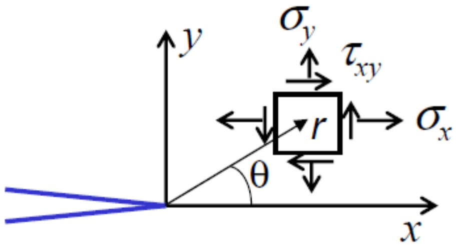
LINEAR ELASTIC FRACTURE MECHANICS

- Stress Intensity Factors (SIFs) are associated to three crack opening modes:



Mode I	Mode II	Mode III
Opening mode	In-plane shear mode	Out-of-plane shear mode
K_I	K_{II}	K_{III}
$K_I = \lim_{r \to 0} \sigma_{yy} \sqrt{2 \pi r}$	$K_{II} = \lim_{r \to 0} \tau_{xy} \sqrt{2 \pi r}$	$K_{III} = \lim_{r \to 0} \tau_{yz} \sqrt{2 \pi r}$

LINEAR ELASTIC FRACTURE MECHANICS



Mode I
Opening mode
K_I

$$u = 2(1+\nu)\frac{K_I}{E}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(1-2\nu+\sin^2\frac{\theta}{2}\right)$$

$$v = 2(1+\nu)\frac{K_I}{E}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(2-2\nu+\cos^2\frac{\theta}{2}\right)$$

$$w = 0$$

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{3}\sin\frac{3\theta}{2}\right)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{3}\sin\frac{3\theta}{2}\right)$$

$$\sigma_z = \begin{cases}\nu(\sigma_x+\sigma_y) & \text{plane strain}\\ 0 & \text{plane stress}\end{cases}$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\tau_{xz} = \tau_{yz} = 0$$

Mode II
In-plane shear mode
K_{II}

$$u = 2(1+\nu)\frac{K_{II}}{E}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(2-2\nu+\cos^2\frac{\theta}{2}\right)$$

$$v = 2(1+\nu)\frac{K_{II}}{E}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(-1+2\nu+\sin^2\frac{\theta}{2}\right)$$

$$w = 0$$

$$\sigma_x = \frac{-K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{3}\cos\frac{3\theta}{2}\right)$$

$$\sigma_y = \frac{K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\sigma_z = \begin{cases}\nu(\sigma_x+\sigma_y) & \text{plane strain}\\ 0 & \text{plane stress}\end{cases}$$

$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\tau_{xz} = \tau_{yz} = 0$$

Mode III
Out-of-plane shear mode
K_{III}

$$u = v = 0$$

$$w = 2(1+\nu)\frac{K_{III}}{E}\sqrt{\frac{2r}{\pi}}\sin\frac{\theta}{2}$$

$$\sigma_x = \sigma_y = \sigma_z = \tau_{xy} = 0$$

$$\tau_{xz} = \frac{-K_{III}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}$$

$$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}$$

STRESS INTENSITY FACTORS

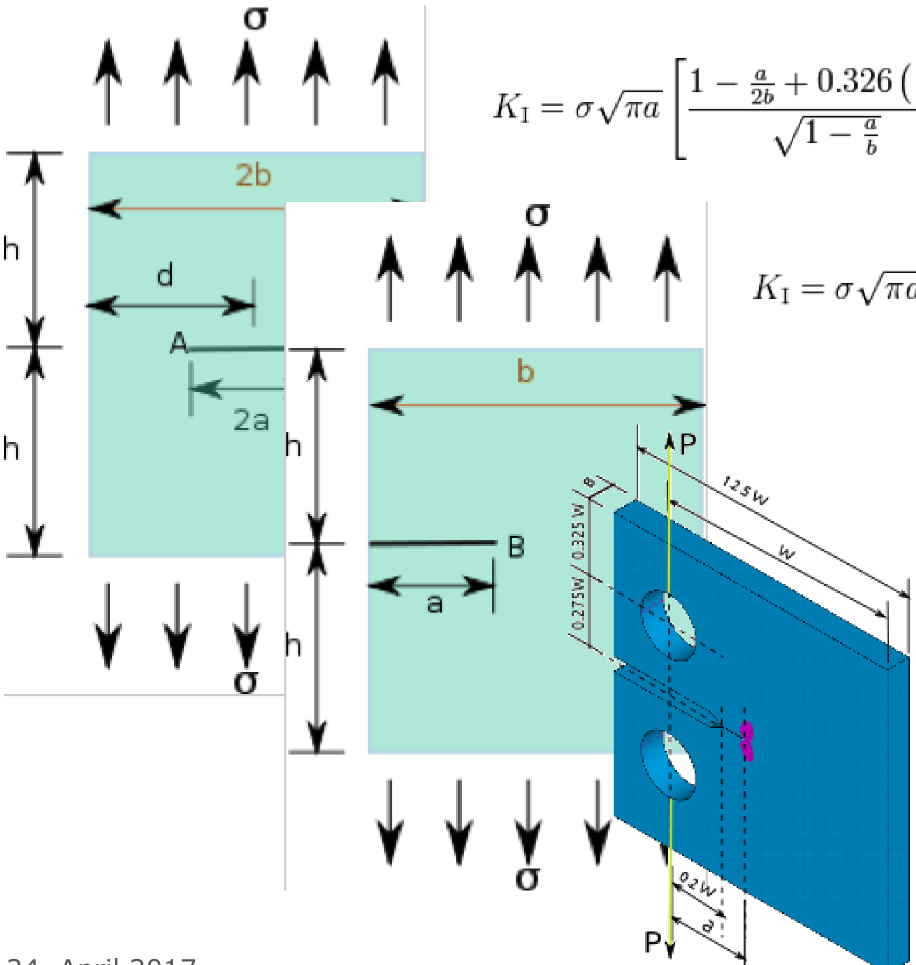
- Stress Intensity Factors must be calculated.

- In general the Opening Mode *KI* is the most relevant (KII and KIII are more involved in the crack kinking/twisting).

- There are multiple ways to obtain *KI*:
    - From Handbooks
    - From FEM (or BEM) calculations
    - From X-FEM calculations
    - With Weight Functions

STRESS INTENSITY FACTORS CALCULATION METHODS

- SIFs from Handbooks (TADA, Rooke-Cartright, …)



$$K_{\mathrm{I}} = \sigma\sqrt{\pi a}\left[\frac{1-\frac{a}{2b}+0.326\left(\frac{a}{b}\right)^2}{\sqrt{1-\frac{a}{b}}}\right]$$

$$K_{\mathrm{I}} = \sigma\sqrt{\pi a}\left[1.12-0.23\left(\frac{a}{b}\right)+10.6\left(\frac{a}{b}\right)^2-21.7\left(\frac{a}{b}\right)^3+30.4\left(\frac{a}{b}\right)^4\right]$$

$$K_{\mathrm{I}} = \frac{P}{B}\sqrt{\frac{\pi}{W}}\left[16.7\left(\frac{a}{W}\right)^{1/2}-104.7\left(\frac{a}{W}\right)^{3/2}+369.9\left(\frac{a}{W}\right)^{5/2} -573.8\left(\frac{a}{W}\right)^{7/2}+360.5\left(\frac{a}{W}\right)^{9/2}\right]$$

STRESS INTENSITY FACTORS CALCULATION METHODS

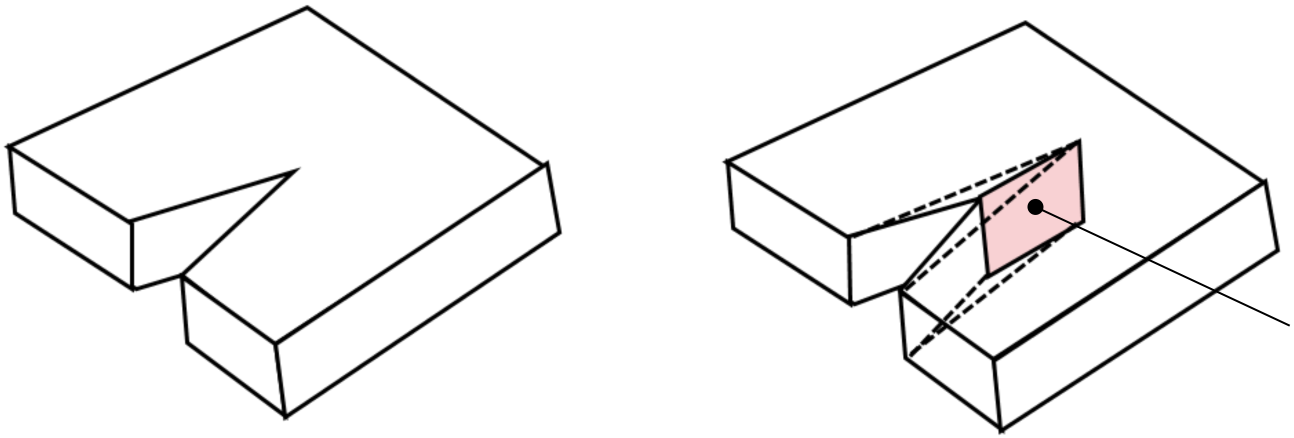
- SIFs from FEM
  - From FEM there are many ways that can be used to calculate *KI*.
    1. From Energy Release rate
    2. With Displacement Correlation
    3. With Virtual Crack Extension Method
    4. With Crack Closure Integral Method
    5. With Modified Crack Closure Integral Method
    6. With J-Integral Method
    7. With M-Interaction Integral Method
    8. ...
  - (1), (2) and (6) are briefly shown.

STRESS INTENSITY FACTORS CALCULATION METHODS

- SIFs from FEM - **Energy Release rate**
  - The Stress Intensity Factors KI (and KII, KIII) in the three modes of fracture are uniquely related to the Energy Release Rate G.
  - $\mathrm{G} = -\frac{\mathrm{d}\Phi}{\mathrm{d}A}$ being Φ the Total Strain Energy, A the crack opening surface.
  - 3D Plane Strain: $G = G_I + G_{II} + G_{III} = \frac{1-v^2}{E}\left({K_I}^2 + {K_{II}}^2\right) + \frac{1+v}{E}{K_{III}}^2$
  - 2D Plane Strain: $G = G_I + G_{II} = \frac{1-v^2}{E}\left({K_I}^2 + {K_{II}}^2\right)$
  - Plane Stress: $G = G_I + G_{II} = \frac{1}{E}\left({K_I}^2 + {K_{II}}^2\right)$

STRESS INTENSITY FACTORS CALCULATION METHODS

- SIFs from FEM - **Energy Release rate**
  - From FEM the Total Strain Energy Φ is calculated at each crack propagation step.



Step i

Calculation of Φ_i

Step i+1

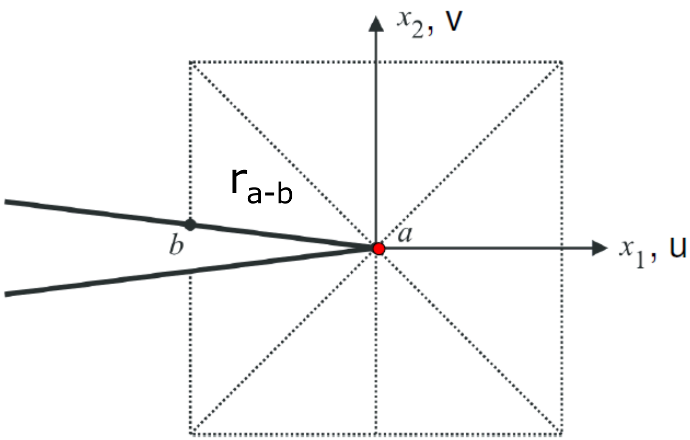
Calculation of Φ_{i+1}

$$G = -\frac{d\Phi}{dA} = \frac{\Phi_i - \Phi_{i+1}}{dA} = \frac{\Phi_i - \Phi_{i+1}}{t \cdot da} = \frac{1-v^2}{E}\left({K_I}^2 + {K_{II}}^2\right) \quad or \quad \frac{1}{E}\left({K_I}^2 + {K_{II}}^2\right)$$

Plane strain Plane stress

STRESS INTENSITY FACTORS CALCULATION METHODS

- SIFs from FEM – **Displacement Correlation**
  - The idea is simple: correlate computed (FEM/BEM) local displacements with their theoretical values, with SIF as scaling parameter.



$$u = \frac{K_I}{\mu}\left[\frac{r}{2\pi}\right]^{1/2}\cos\frac{\theta}{2}\left[1-2\nu+\sin^2\frac{\theta}{2}\right]$$

$$v = \frac{K_I}{\mu}\left[\frac{r}{2\pi}\right]^{1/2}\sin\frac{\theta}{2}\left[2-2\nu-\cos^2\frac{\theta}{2}\right] \longrightarrow v_b - v_a = \frac{K_I}{\mu}\left[\frac{r_{a-b}}{2\pi}\right]^{1/2}(2-2\nu)$$

where μ is the shear modulus

Set $r = r_{a-b}$, and $\theta = 180°$

Note: for plane stress, let $\nu = \nu/(1+\nu)$

$$u = \frac{K_{II}}{\mu}\left[\frac{r}{2\pi}\right]^{1/2}\sin\frac{\theta}{2}\left[2-2\nu+\cos^2\frac{\theta}{2}\right] \longrightarrow u_b - u_a = \frac{K_{II}}{\mu}\left[\frac{r_{a-b}}{2\pi}\right]^{1/2}(2-2\nu)$$

$$v = \frac{K_{II}}{\mu}\left[\frac{r}{2\pi}\right]^{1/2}\cos\frac{\theta}{2}\left[-1+2\nu+\sin^2\frac{\theta}{2}\right]$$

For plane strain case: $K_I = \dfrac{\mu\sqrt{2\pi}(v_b - v_a)}{\sqrt{r_{a-b}}(2-2\nu)}$ $\quad K_{II} = \dfrac{\mu\sqrt{2\pi}(u_b - u_a)}{\sqrt{r_{a-b}}(2-2\nu)}$

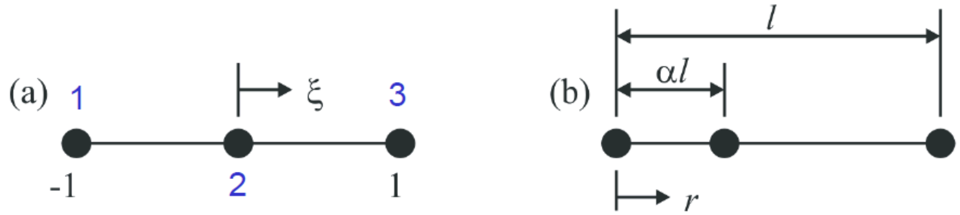
The same expressions can be used for plane stress assumptions if ν is replaced with $\nu = \nu/(1+\nu)$.

STRESS INTENSITY FACTORS CALCULATION METHODS

- SIFs from FEM
  - The stress field at the crack tip is singular.
  - The singularity makes the problem not compatible with standard FEM-BEM polynomial-based formulations.
  - If one wants to reproduce these fields with FEM-BEM, many small elements are needed, especially in the K-dominant region.
  - However, in the limit as one approaches the crack front, we can never reproduce the singular nature with polynomial based elements.
  - This makes calculating SIF's accurately using local field information difficult and inefficient with such elements, however…

STRESS INTENSITY FACTORS CALCULATION METHODS

- SIFs from FEM
  - …Quarter point elements reproduces the correct leading displacements and strain terms.



A quadratic element, (a) the parametric space of the element, (b) the Cartesian space of the element. The crack tip is at r=0.

$$u=\sum_{i=1}^{3} N_i u_i = \tfrac{1}{2}\xi(\xi-1)u_1 + (1-\xi^2)u_2 + \tfrac{1}{2}\xi(\xi+1)u_3 \longrightarrow u = u_2 + \tfrac{1}{2}(u_3-u_1)\xi + (\tfrac{1}{2}(u_1+u_3)-u_2)\xi^2$$

$$r=\sum_{i=1}^{3} N_i r_i = \alpha l + \tfrac{1}{2} l\xi + l(\tfrac{1}{2}-\alpha)\xi^2$$

Its standard, polynomial *geometry* interpolation scheme.

First, the usual case of mid-side geometry:

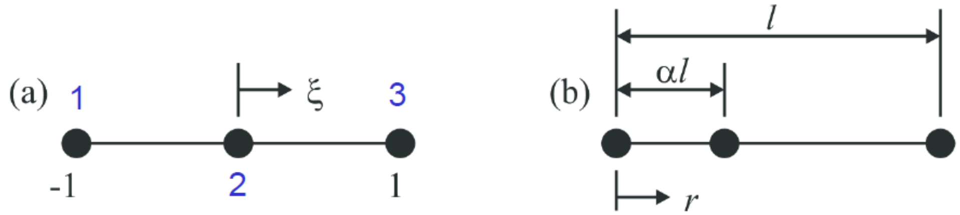
$$\alpha = \frac{1}{2} \longrightarrow \xi = \frac{2r}{l} - 1$$

Then we get the expected, polynomial interpolation:

$$u = u_1 + (-3u_1 + 4u_2 - u_3)\frac{r}{l} + 2(u_1 - 2u_2 + u_3)\frac{r^2}{l^2}$$

STRESS INTENSITY FACTORS CALCULATION METHODS

- SIFs from FEM
  - …Quarter point elements reproduces the correct leading displacements and strain terms.



A quadratic element, (a) the parametric space of the element, (b) the Cartesian space of the element. The crack tip is at r=0.

$$u=\sum_{i=1}^{3}N_i u_i=\tfrac{1}{2}\xi(\xi-1)u_1+(1-\xi^2)u_2+\tfrac{1}{2}\xi(\xi+1)u_3 \longrightarrow u=u_2+\tfrac{1}{2}(u_3-u_1)\xi+(\tfrac{1}{2}(u_1+u_3)-u_2)\xi^2$$

$$r=\sum_{i=1}^{3}N_i r_i=\alpha l+\tfrac{1}{2}l\xi+l(\tfrac{1}{2}-\alpha)\xi^2$$

Its standard, polynomial *geometry* interpolation scheme.

Next, the unusual case of ¼-point geometry:

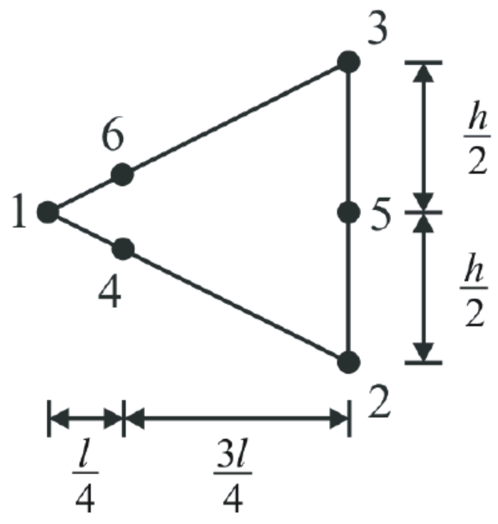
$$\alpha=\frac{1}{4} \longrightarrow \xi=\frac{2\sqrt{lr}}{l}-1$$

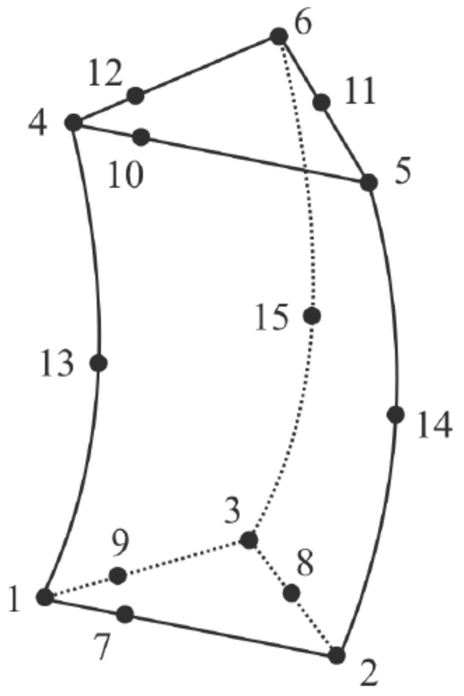
Then we get the *unexpected*, non-polynomial interpolation !!!:

$$u=u_1+2(u_1-2u_2+u_3)\frac{r}{l}+(-3u_1+4u_2+u_3)\frac{\sqrt{lr}}{l}$$

STRESS INTENSITY FACTORS CALCULATION METHODS

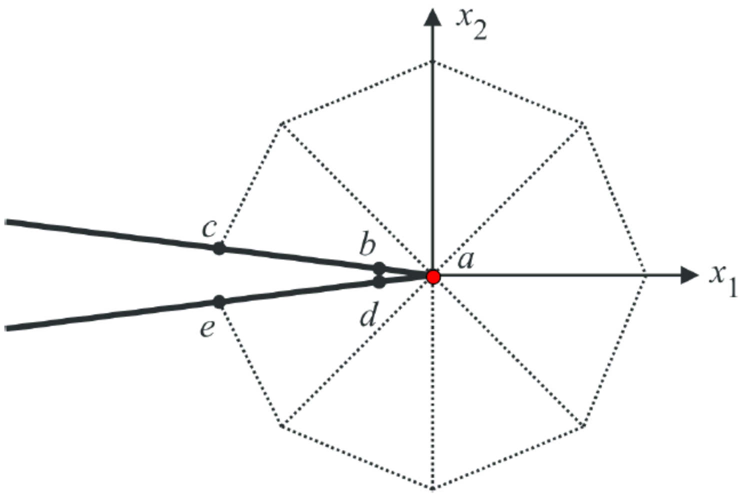
- SIFs from FEM





STRESS INTENSITY FACTORS CALCULATION METHODS

- SIFs from FEM – **Displacement Correlation**
  - If ¼-point elements are used:



Write the displacement function along $r_{a\text{-}b\text{-}c}$ and along $r_{a\text{-}d\text{-}e}$

$$v_{upper} = v_a + (-3v_a + 4v_b - v_c)\sqrt{\frac{r}{l}} + (2v_a - 4v_b + 2v_c)\frac{r}{l}$$

$$v_{lower} = v_a + (-3v_a + 4v_d - v_e)\sqrt{\frac{r}{l}} + (2v_a - 4v_d + 2v_e)\frac{r}{l}$$

$$v_{upper} - v_{lower} = [4(v_b - v_d) + v_e - v_c]\sqrt{\frac{r}{l}} + [4(v_b - v_d) + 2(v_c - v_e)]\frac{r}{l}$$

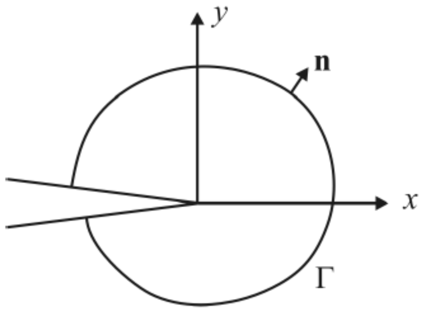
$$K_I = \frac{\mu\sqrt{2\pi}}{\sqrt{r_{a-b-c}}(2-2\nu)}[4(v_b - v_d) + v_e - v_c]$$

$$u_{upper} - u_{lower} = [4(u_b - u_d) + u_e - u_c]\sqrt{\frac{r}{l}} + [4(u_b - u_d) + 2(u_c - u_e)]\frac{r}{l}$$

$$K_{II} = \frac{\mu\sqrt{2\pi}}{\sqrt{r_{a-b-c}}(2-2\nu)}[4(u_b - u_d) + u_e - u_c]$$

STRESS INTENSITY FACTORS CALCULATION METHODS

- SIFs from FEM – **J-Integral (Rice, 1968)**

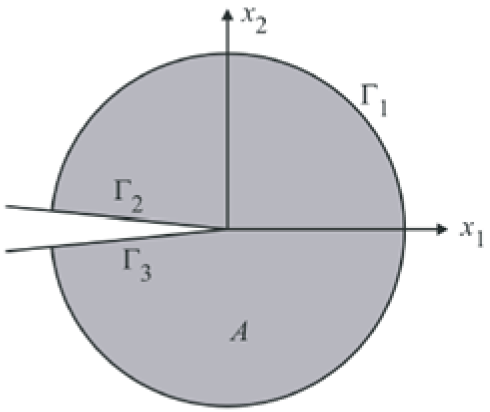


$$J = \int_\Gamma \left(W n_x - T_i \frac{\partial u_i}{\partial x} \right) ds$$

  - Where $W = \frac{1}{2}\sigma_{ij}\varepsilon_{ij}$ is the strain energy density, *T* is the surface traction vector, *n* is the unit outward normal to the contour, *u* is the displacement vector.
  - The *J*-Integral measures the energy flux into the crack-tip region.
  - Rice demonstrated that; **(1) under small scale yielding conditions, the *J*-Integral is equal to the Energy Release Rate, G**….

STRESS INTENSITY FACTORS CALCULATION METHODS

- SIFs from FEM – **J-Integral (Rice, 1968)**
  - **…(2) for a material which is characterized by linear or nonlinear elastic behavior, *J* is path-independent.**
  - The contour *J*-Integral can be recast as an equivalent area (volume in 3D) integral (invoking divergence theorem), which is more accurate and stable in a finite element context.



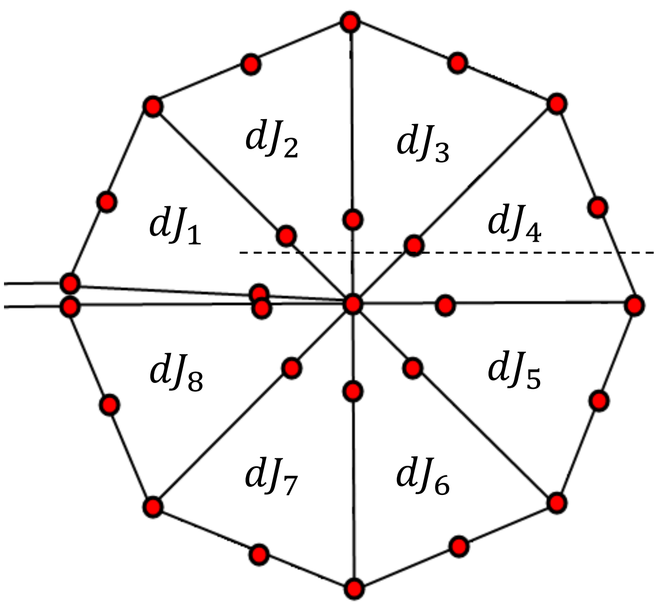
$$\bar{J} = \int_A \left[\sigma_{ij} \frac{\partial u_i}{\partial x_1} - W\delta_{1j} \right] \frac{\partial q_1}{\partial x_j} \, dA \qquad q = \sum_i N_i q_i \qquad \frac{\partial q}{\partial x_j} = \sum_i \frac{\partial N_i}{\partial x_j} q_i$$

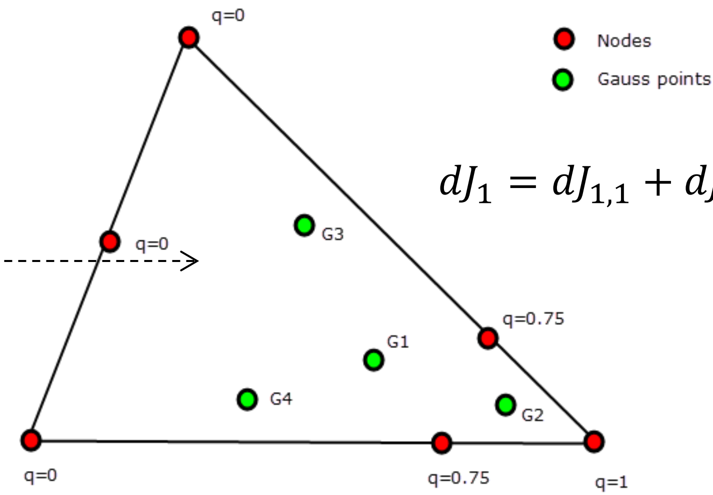
Where:

- δ is the Kroneker delta.
- *N* is the element shape function.
- *q* is a weighting function defined over the domain of integration (=1 at the crack tip, =0 on the boundary of the integration domain, =0.75 at the ¼ points).

STRESS INTENSITY FACTORS CALCULATION METHODS

- SIFs from FEM – **J-Integral (Rice, 1968)**





$$dJ_1 = dJ_{1,1} + dJ_{1,2} + dJ_{1,3} + dJ_{1,4}$$

$$J = \sum_{i=1}^{N_{elms}} \left(\sum_{j=1}^{4} dJ_{i,j} \right)$$

where i is the elements counter, j is the Gauss points counter

STRESS INTENSITY FACTORS CALCULATION METHODS

- SIFs from FEM – **J-Integral (Rice, 1968)**

$$dJ_{i,j} = \left(\sigma_{i,j}\frac{\partial u_i}{\partial x_1} - W\delta_{1j}\right)\frac{\partial q}{\partial x_j}dA_j = \left[\left(\sigma_{11}\frac{\partial u}{\partial x} + \sigma_{12}\frac{\partial v}{\partial x} - W\right)\frac{\partial q}{\partial x} + \left(\sigma_{12}\frac{\partial u}{\partial x} + \sigma_{22}\frac{\partial v}{\partial x}\right)\frac{\partial q}{\partial y}\right]dA_j$$

$$W = \frac{1}{2}(\sigma_{11}\sigma_{11} + \sigma_{22}\sigma_{22} + \sigma_{12}\sigma_{12})$$

$$\frac{\partial q}{\partial x} = \sum_{k=1}^{6}\frac{\partial N_{k,1}}{\partial x}q_k \qquad \frac{\partial q}{\partial y} = \sum_{k=1}^{6}\frac{\partial N_{k,2}}{\partial y}q_k \qquad dA_j = \det[J]\cdot w_j$$

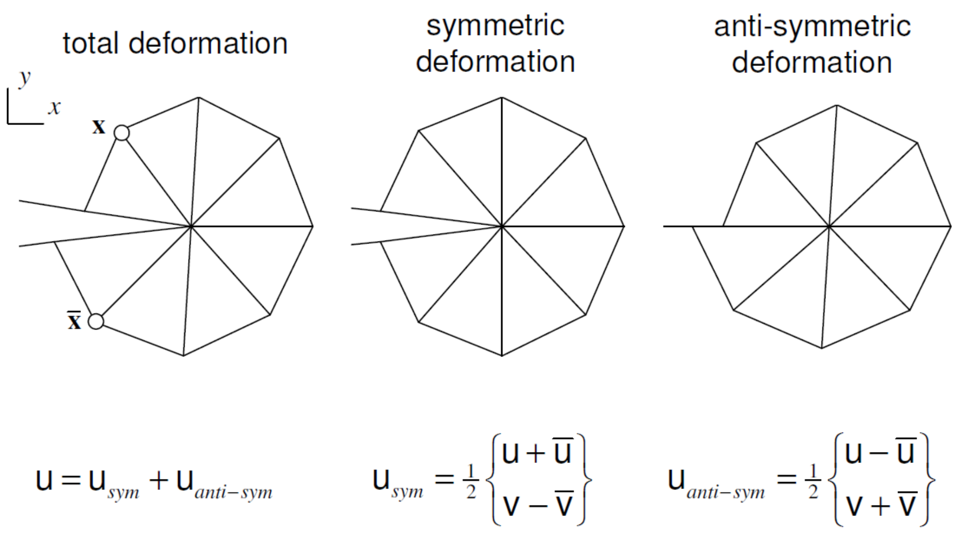
$$\frac{\partial N_k}{\partial x} = \frac{J_{22}}{\det[J]}\frac{\partial N_{k,1}}{\partial s} - \frac{J_{21}}{\det[J]}\frac{\partial N_{k,1}}{\partial t} \qquad \frac{\partial N_k}{\partial x} = -\frac{J_{12}}{\det[J]}\frac{\partial N_{k,2}}{\partial s} + \frac{J_{11}}{\det[J]}\frac{\partial N_{k,2}}{\partial t}$$

Being $[J] = \begin{bmatrix}\frac{\partial x}{\partial s} & \frac{\partial y}{\partial s}\\ \frac{\partial x}{\partial t} & \frac{\partial y}{\partial t}\end{bmatrix} = \begin{bmatrix}\sum_{k=1}^{6}\frac{\partial N_{k,1}}{\partial s}x_k & \sum_{k=1}^{6}\frac{\partial N_{k,2}}{\partial s}y_k\\ \sum_{k=1}^{6}\frac{\partial N_{k,1}}{\partial t}x_k & \sum_{k=1}^{6}\frac{\partial N_{k,2}}{\partial t}y_k\end{bmatrix}$ the element Jacobian matrix

$[N](s,t)$ = the element shape functions matrix; *s* and *t* the node k coordinates in the element mapped space.

STRESS INTENSITY FACTORS CALCULATION METHODS

- SIFs from FEM – **J-Integral (Rice, 1968)**
  - 2D Plane Strain condition: $J = G = \frac{1-\nu^2}{E}\left(K_I{}^2 + K_{II}{}^2\right)$
  - 2D Plane Stress condition: $J = G = \frac{1}{E}\left(K_I{}^2 + K_{II}{}^2\right)$



- Separate the modes by decomposing the near crack-tip displacement fields into one field that is symmetric with respect to the crack and another field that is anti-symmetric with respect to the crack.

STRESS INTENSITY FACTORS CALCULATION METHODS

- SIFs from FEM – **J-Integral (Rice, 1968)**

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_{sym} + \boldsymbol{\sigma}_{anti-sym} = \tfrac{1}{2}\begin{bmatrix} \sigma_{11} + \bar{\sigma}_{11} & \sigma_{12} - \bar{\sigma}_{12} \\ sym & \sigma_{22} + \bar{\sigma}_{22} \end{bmatrix} + \tfrac{1}{2}\begin{bmatrix} \sigma_{11} - \bar{\sigma}_{11} & \sigma_{12} + \bar{\sigma}_{12} \\ sym & \sigma_{22} - \bar{\sigma}_{22} \end{bmatrix}$$

$$G_I = J_I = J(\mathrm{u}_{sym}, \boldsymbol{\sigma}_{sym})$$

$$G_{II} = J_{II} = J(\mathrm{u}_{anti-sym}, \boldsymbol{\sigma}_{anti-sym})$$

$$K_I = \sqrt{\frac{EG_I}{\kappa}} \qquad K_{II} = \sqrt{\frac{EG_{II}}{\kappa}}$$

$$\kappa = \begin{cases} 1 & \text{plane stress} \\ 1-\nu^2 & \text{plane strain} \end{cases}$$

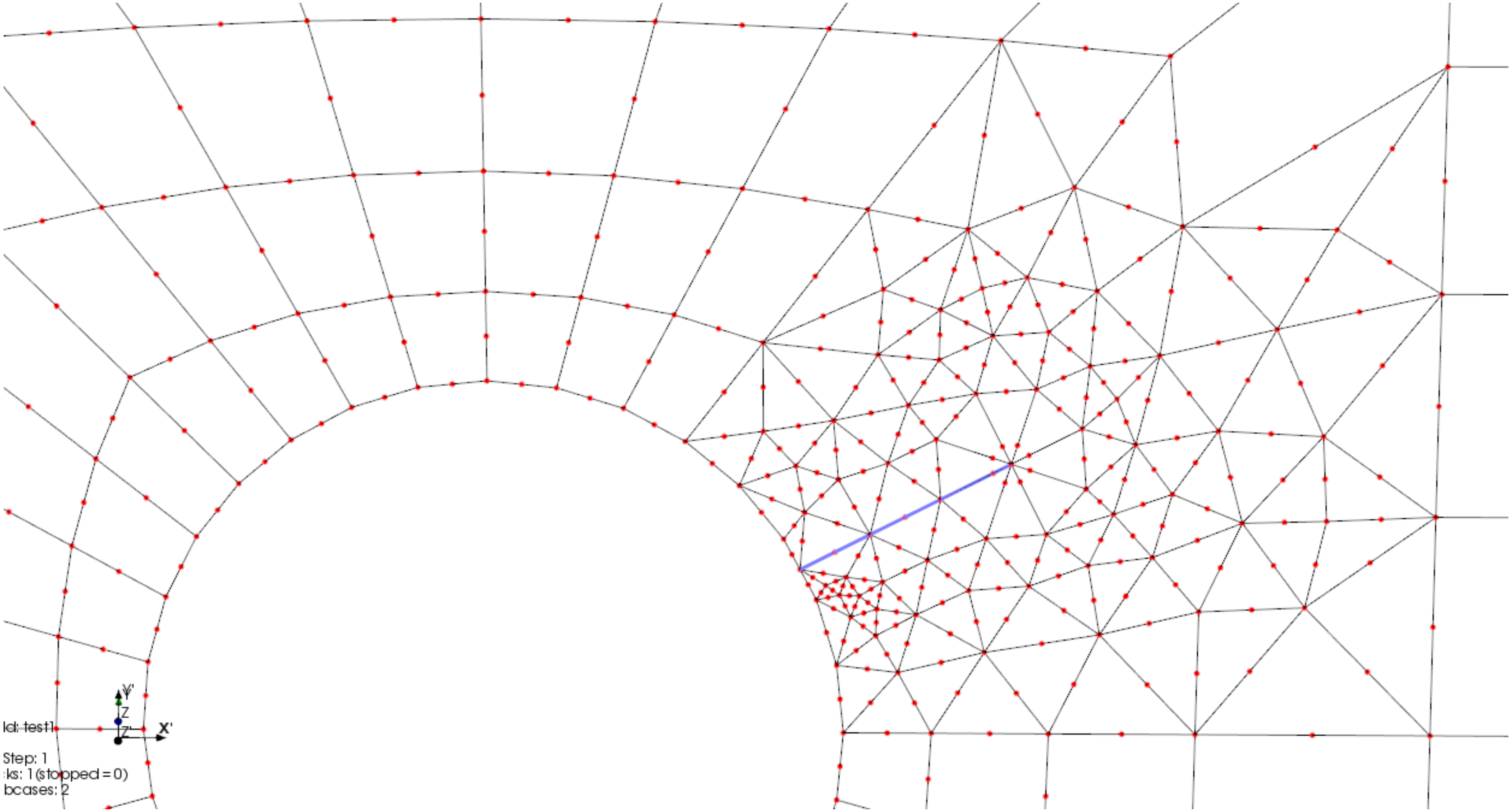
$$J_I, J_{II} \neq J_1, J_2$$

note: $J_1 = K_I^2 + K_{II}^2$

$$J_2 = K_I K_{II}$$

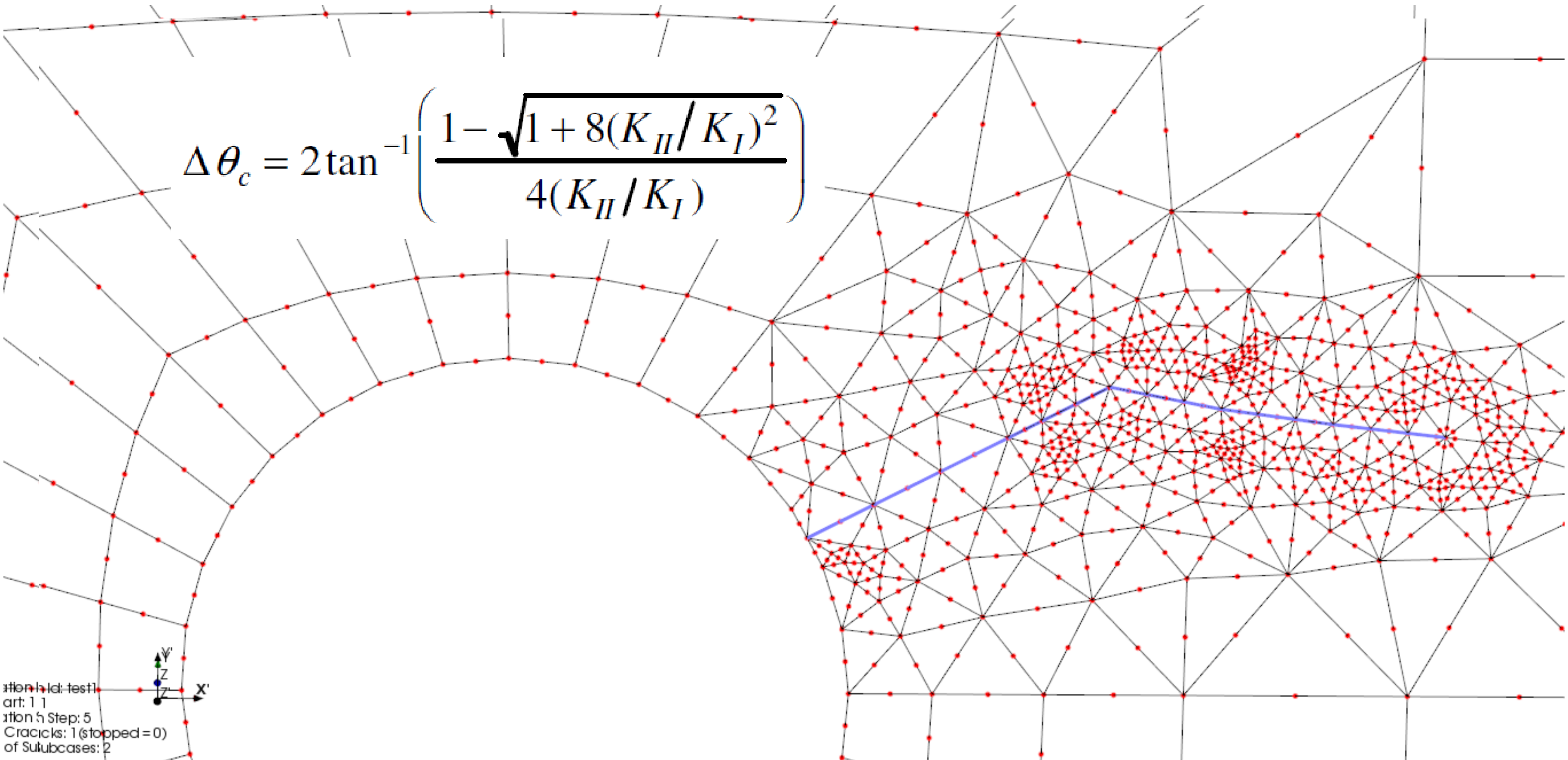
STRESS INTENSITY FACTORS CALCULATION METHODS

- Example with the software Lifing



STRESS INTENSITY FACTORS CALCULATION METHODS

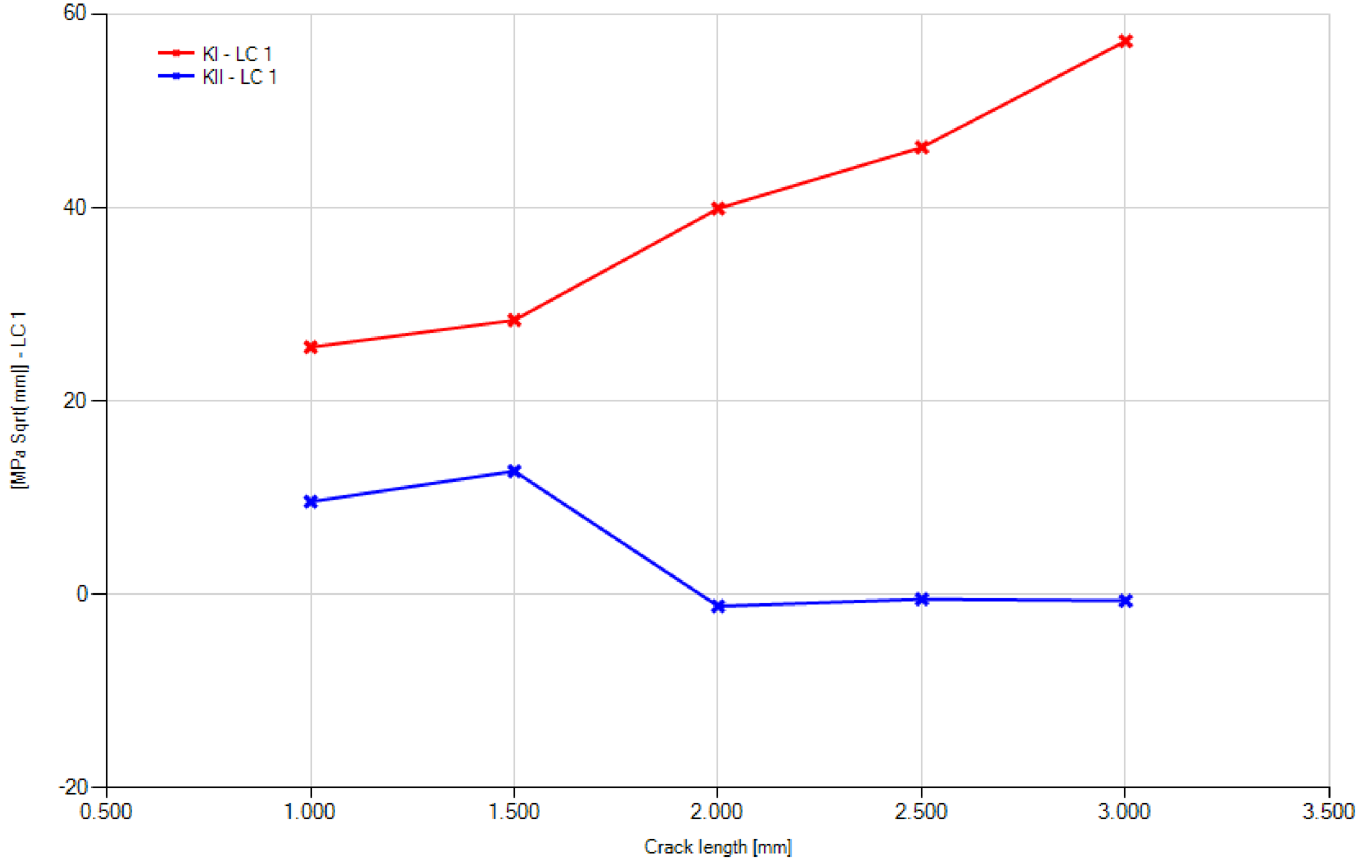
- Example with the software Lifing



$$\Delta\theta_c = 2\tan^{-1}\left(\frac{1-\sqrt{1+8(K_{II}/K_I)^2}}{4(K_{II}/K_I)}\right)$$

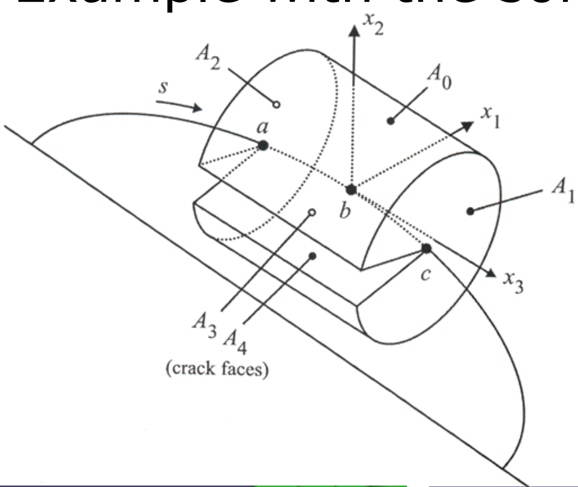
STRESS INTENSITY FACTORS CALCULATION METHODS

- Example with the software Lifing

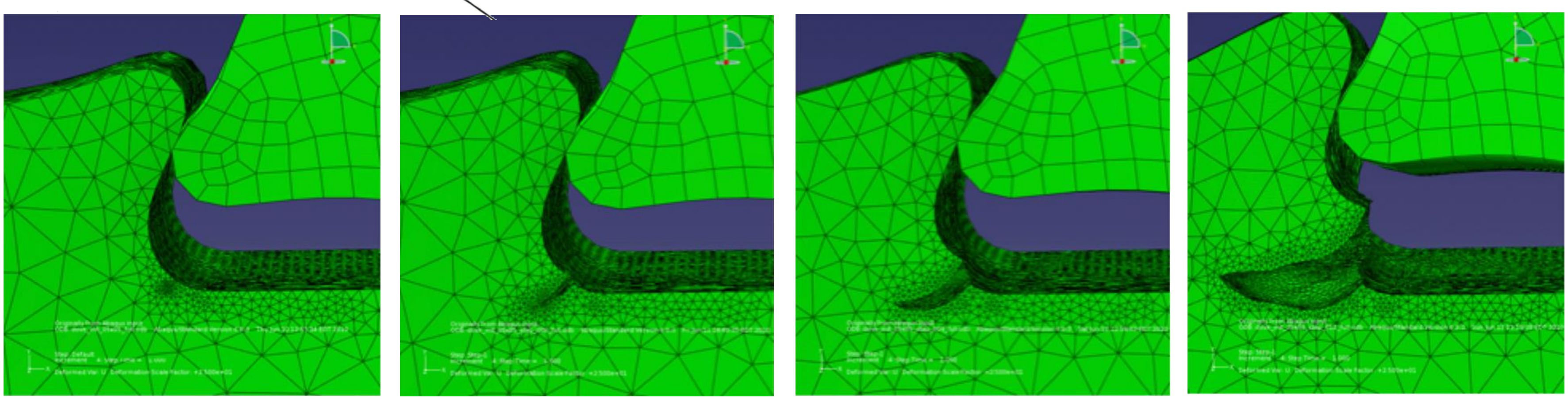


STRESS INTENSITY FACTORS CALCULATION METHODS

- Example with the software Franc3D



$$J = \int_V \left[\sigma_{ij} \frac{\partial u_i}{\partial x_1} - W\delta_{kj} \right] \frac{\partial q_j}{\partial x_k} \, dV + \int_{A_3 + A_4} T_i \frac{\partial u_i}{\partial x_1} q \, dA$$



STRESS INTENSITY FACTORS CALCULATION METHODS

Business Case

- Running this kind of analysis IS NOT CHEAP but the cost besnefit can be HUGE
- Example:
  - Assume we have a fleet of **500** Aircraft
  - Assume that a structural detail (analysed with conventional Analysis Methods) has to be inspected **once every year after 10** years of Service
  - Assume that the fleet has to be in Service for **other 10 years**
  - Assume each inspection (NDI Inspection) has a cost of **500 Euro**
  - The Inspection Programme costs 500x500x10 = **2’500’000 Euro**
  - **...plus the cost of the grounded fleet!!!!**

STRESS INTENSITY FACTORS CALCULATION METHODS

Business Case

- Assume that an analysis like the one in the previous slide takes **300 hr**
- Assume an **hourly rate 100 Euro**
- Analysis cost: 300x100 = **30’000 Euro**
- Assume that with this analysis we are able to relax the Inspection Interval to 1 Inspection every 2 years (**halved interval**)
- The new Inspection Programme now costs **1’250’000 Euro**
- **Savings**: 1’250’000 + 30’000 – 2’500’000 =

(-) 1’220’000 Euro

CRACK GROWTH MODELS

- Crack Growth Life analysis is the time (or number of cycles) to propagate a flaw of an assumed or measured initial size to a critical dimension.

- In most metallic materials, catastrophic failure is preceded by a substantial amount of stable crack propagation under cyclic loading conditions.

- The rate of growth of a fatigue crack subjected to a constant amplitude stress reversals is expressed in terms of the crack length increment per cycle, da/dN.

CRACK GROWTH MODELS

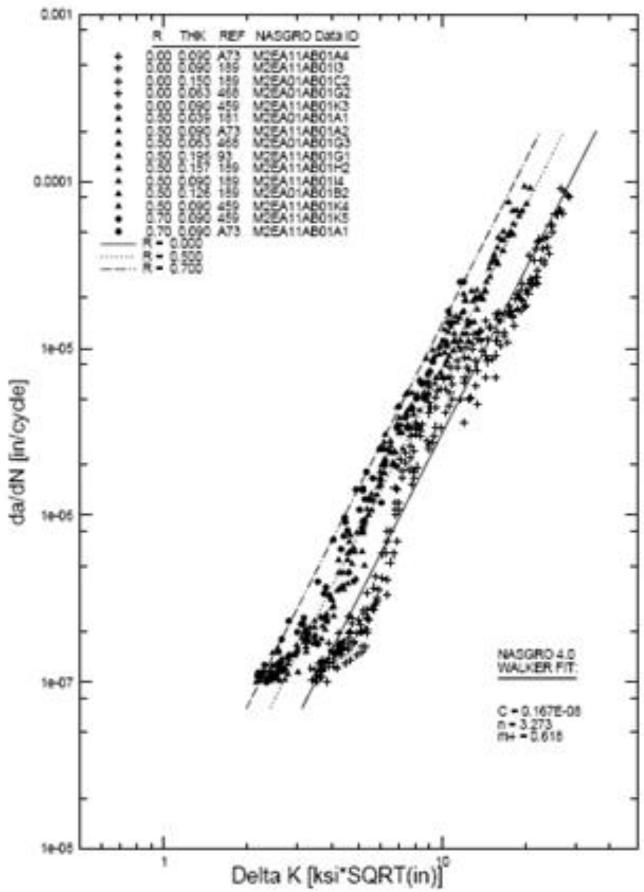
- For a cyclic variation of imposed stress field, the linear elastic fracture mechanics characterization of the rate of fatigue crack growth is based on the SIF range

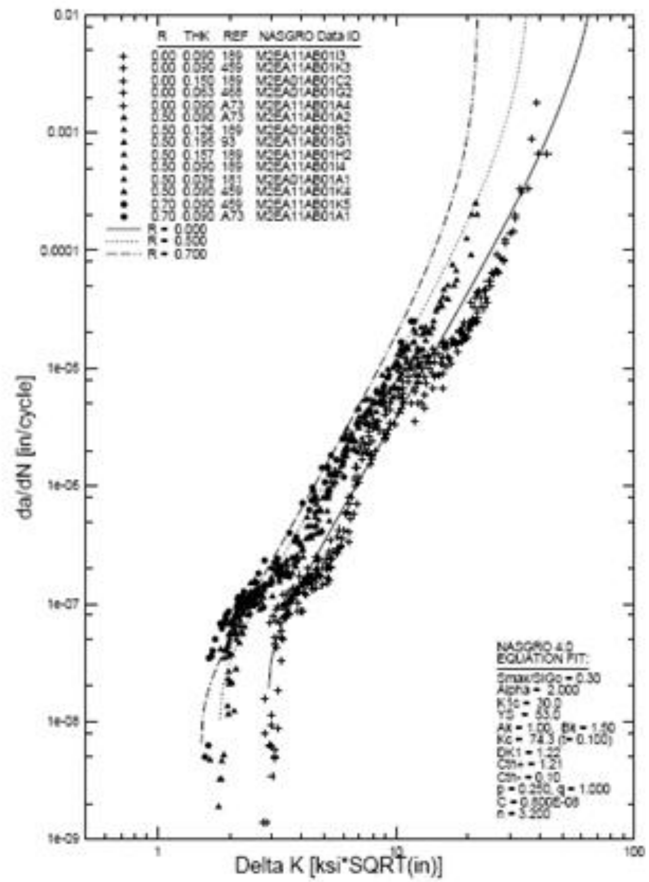
$$\Delta K_I = K_{I,max} - K_{I,min}$$

- Being $K_{I,max}$ and $K_{I,min}$ are the maximum and minimum values, respectively, of SIFs during a fatigue stress cycle.

- Crack Growth Models are in the form da/dN = f(ΔKI, R)

CRACK GROWTH MODELS



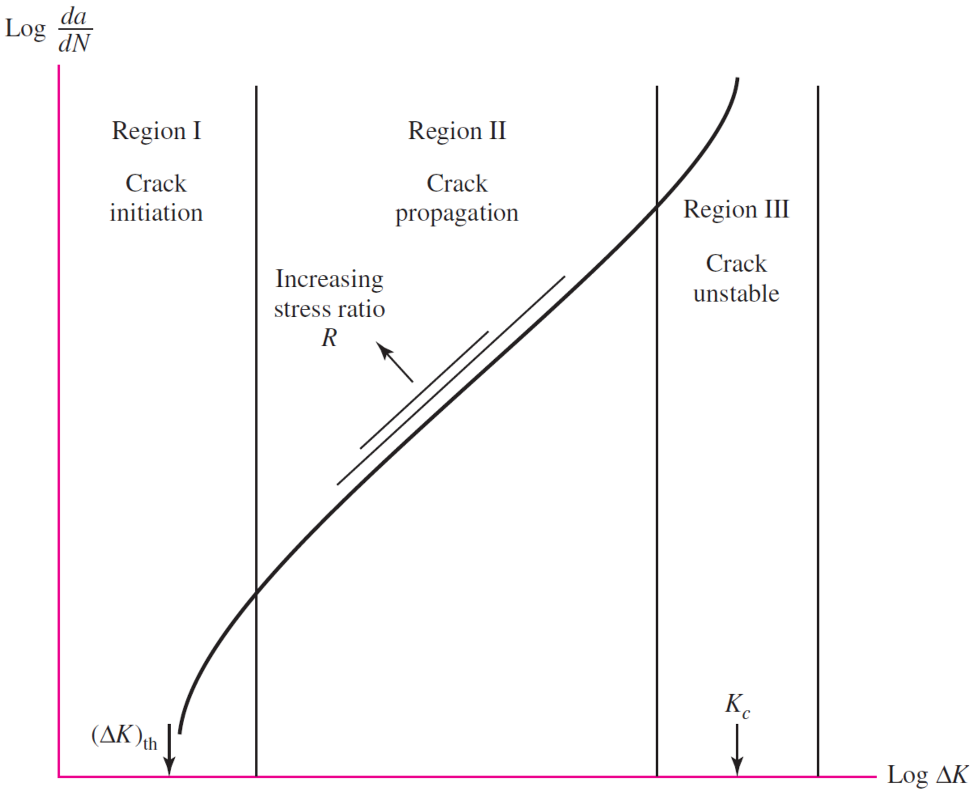


CRACK GROWTH MODELS

- Paris Model

$$\frac{da}{dN} = C(\Delta K)^n$$

  - There is a linear relationship between da/dN and ΔK in a log-log space.
  - C and n are material constant, dependant on environmental conditions and load ratio R

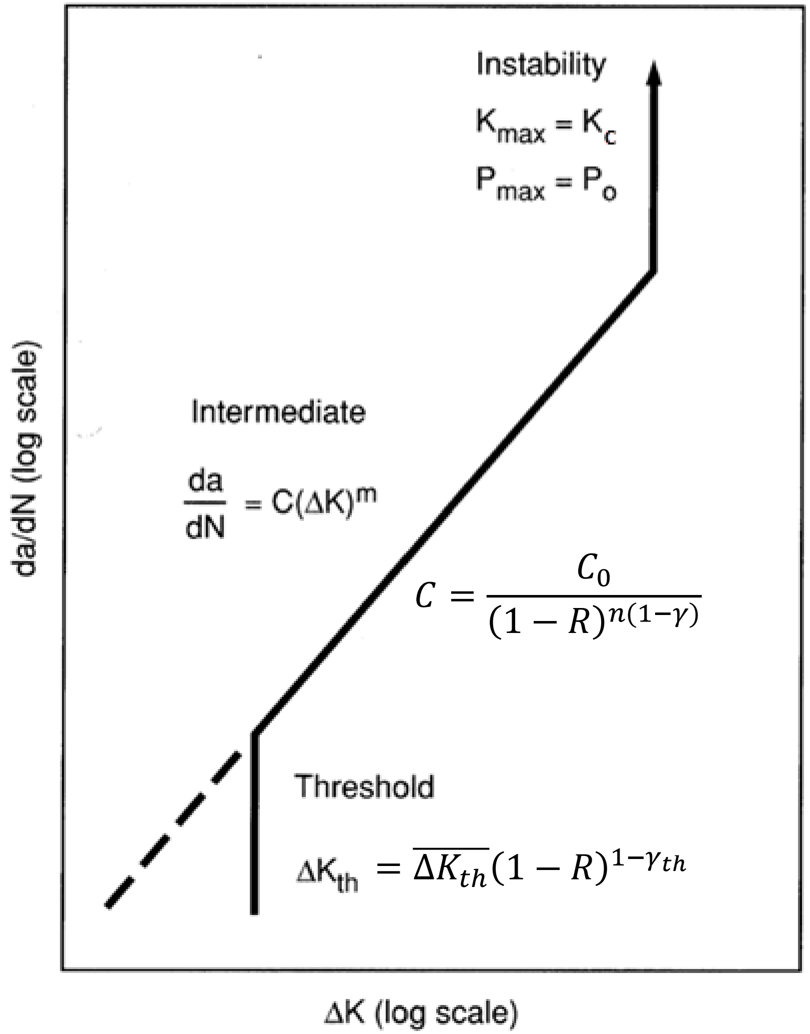


CRACK GROWTH MODELS

- Walker Model

$$\frac{da}{dN} = C(\Delta K)^n$$

  - For simplicity reasons the complete fatigue crack growth rate is usually approximated by three piece curve with the two vertical limiting asymptotes mentioned earlier.

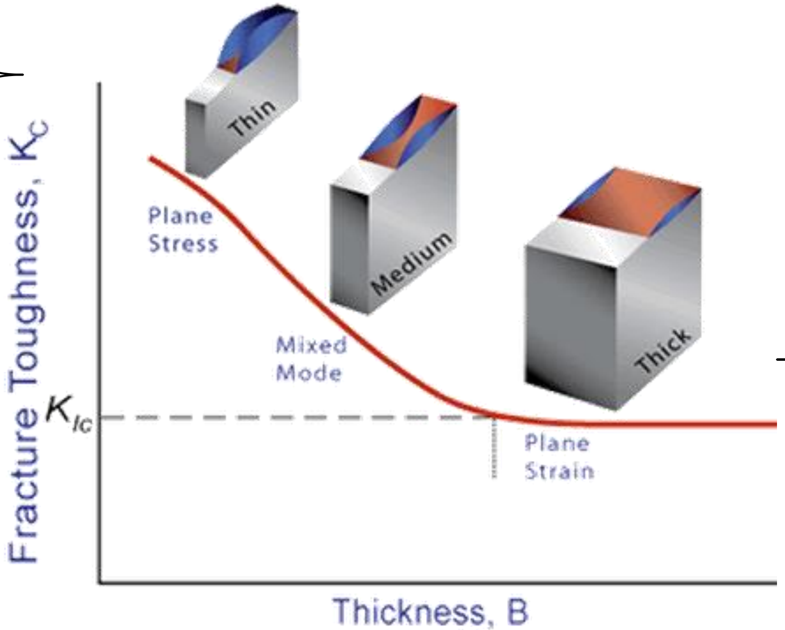


CRACK GROWTH MODELS

- Fracture toughness K_C is a property which describes the ability of a structure containing a crack to resist fracture.
- K_C is a structural property as well, not only a material property: the dimensions (e.g. thickness) influence K_C.
- K_C is derived from the Plane Strain Fracture Toughness K_{IC}, where a thickness correction is applied.

Ductile Fracture

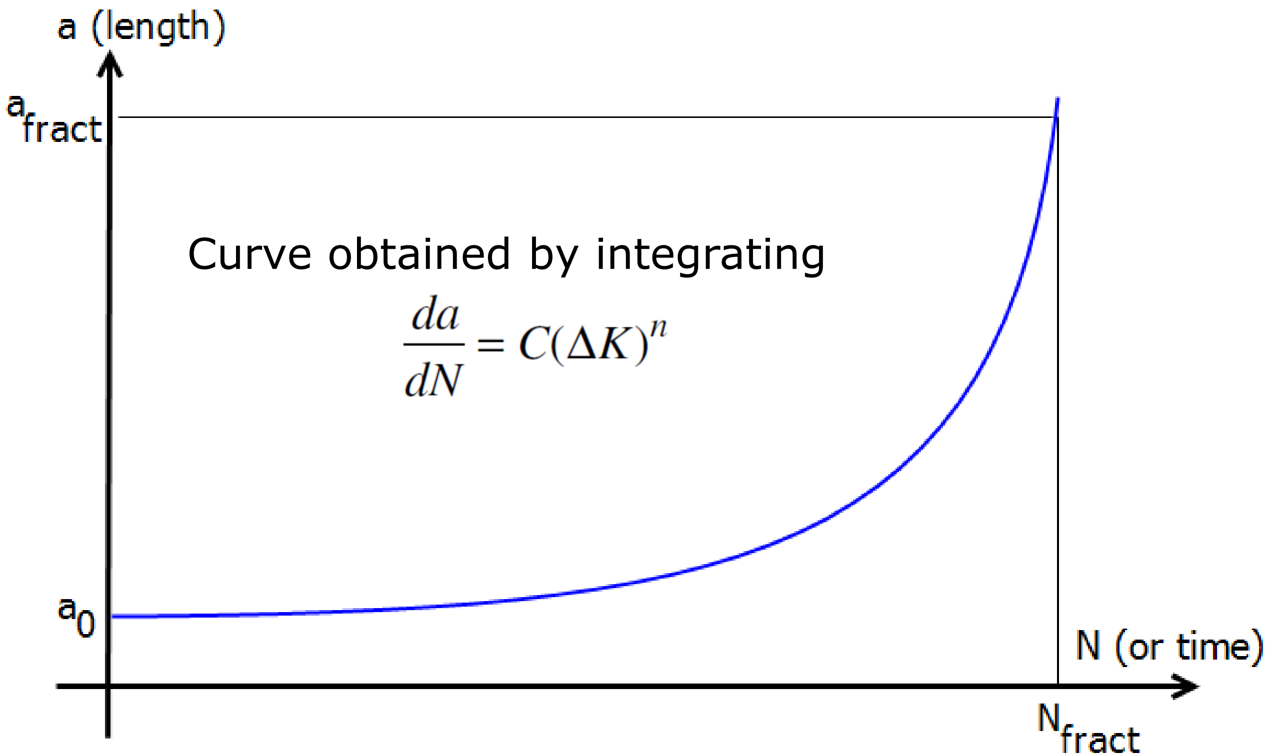
Plastic deformation takes place before fracture



Brittle Fracture

No apparent plastic deformation takes place before fracture

CRACK GROWTH AND RESIDUAL STRENGTH ANALYSIS

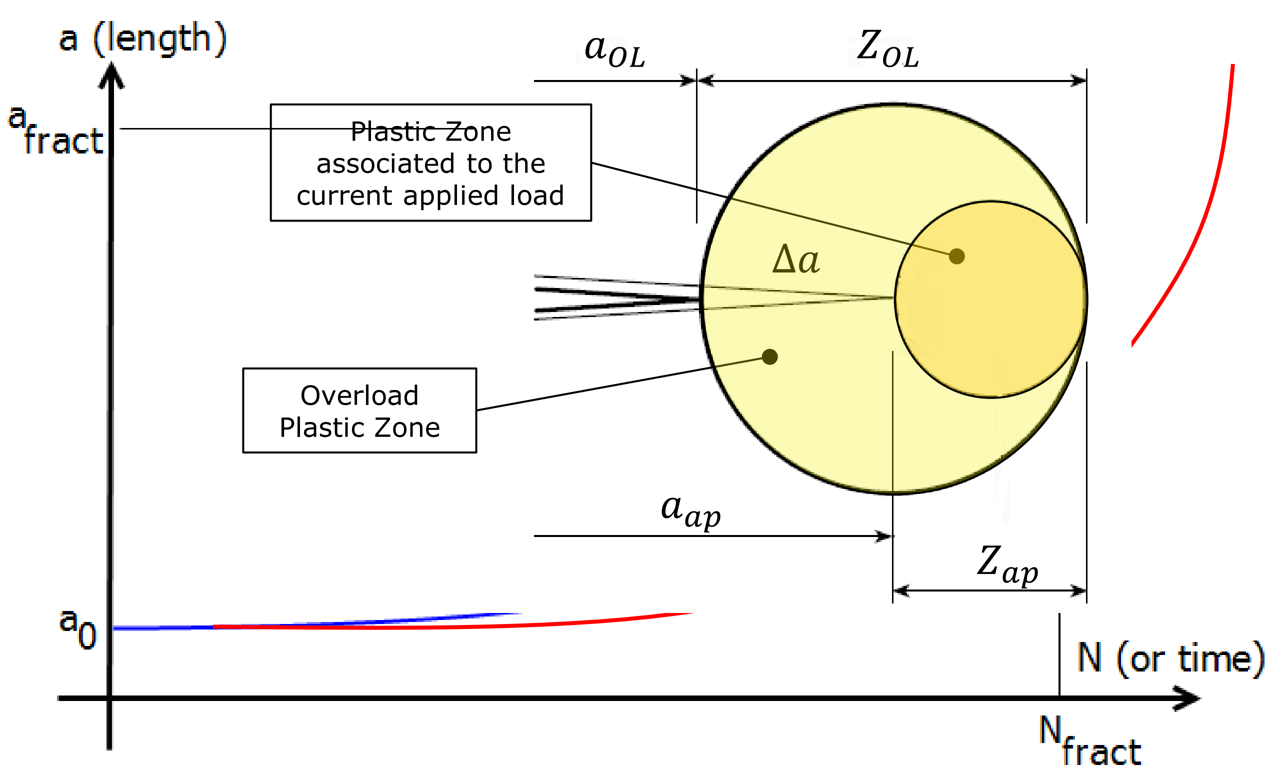


- The analysis starts with a given initial crack a_0 The ‘damage cumulation’ concept looses the meaning described in the Fatigue chapter.
- the crack SIFs change at each cycle because the crack is growing. This is a significant difference wrt to the damage cumulation process employed in fatigue analyses.

- The same stress cycle occurring at different instants contributes differently because the SIF is changed (as the crack has grown). For this reason the time history is integrated multiple times.

CRACK GROWTH AND RESIDUAL STRENGTH ANALYSIS

- Variable amplitude sequence of loads induce 'load interaction' effects which are beneficial (crack retardation).

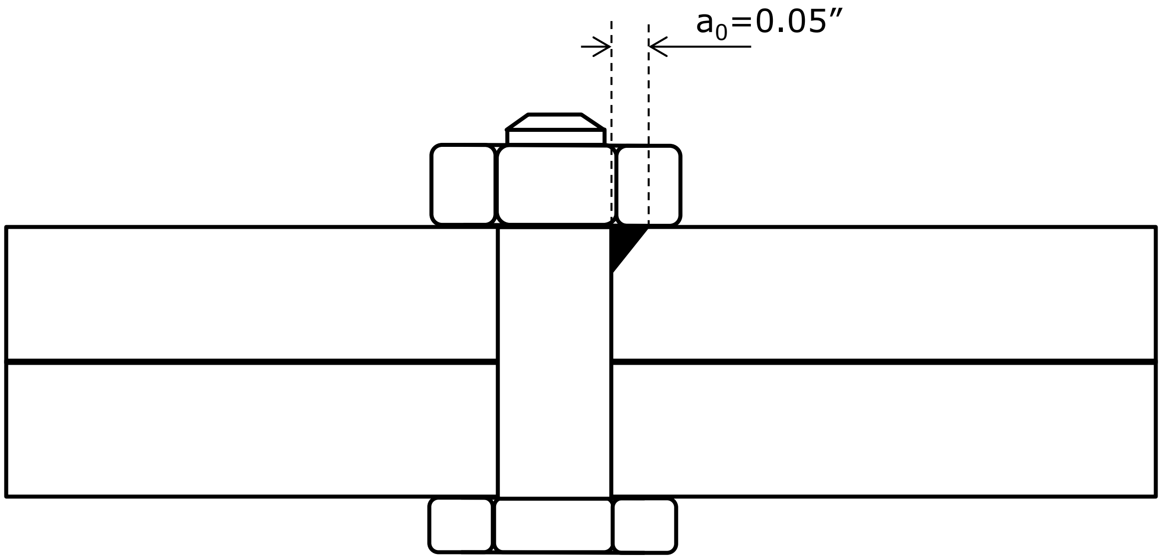


- Local high tension stresses generate compressive residual stresses which tend to close the crack (crack closure).
- the crack keeps propagating when another tension load cycle 'breaks through' the compressive region.
- Available retardation models (Willemborg, Wheeler, Fastran, …) must be calibrated by tests.

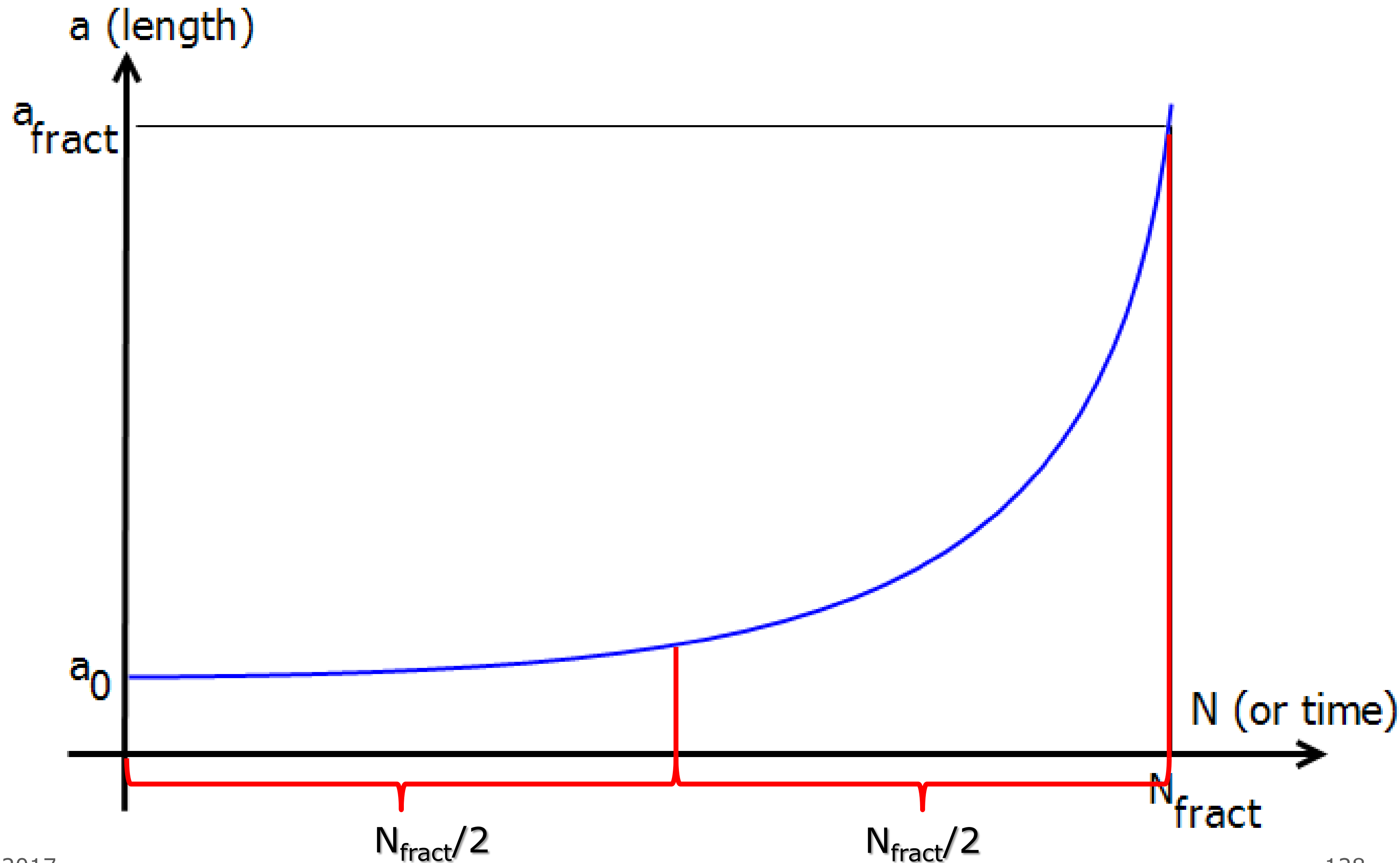
INSPECTION INTERVALS

- Once the CG curve is calculated (life to fracture from an **initial crack size**), this is the principal tool for the definition of Safety by Inspection Regime.
- Two kind of inspections are defined:
  - Threshold inspection: the first inspection to be performed in a given component or area
  - Recurrent inspections: the subsequent ones
- For the definition of these, dedicated Scatter Factors are used
  - Threshold inspection: SF = 2
  - Recurrent inspections: SF = 3 (and a **detectable crack size**, depending on the inspection technique, must be defined)

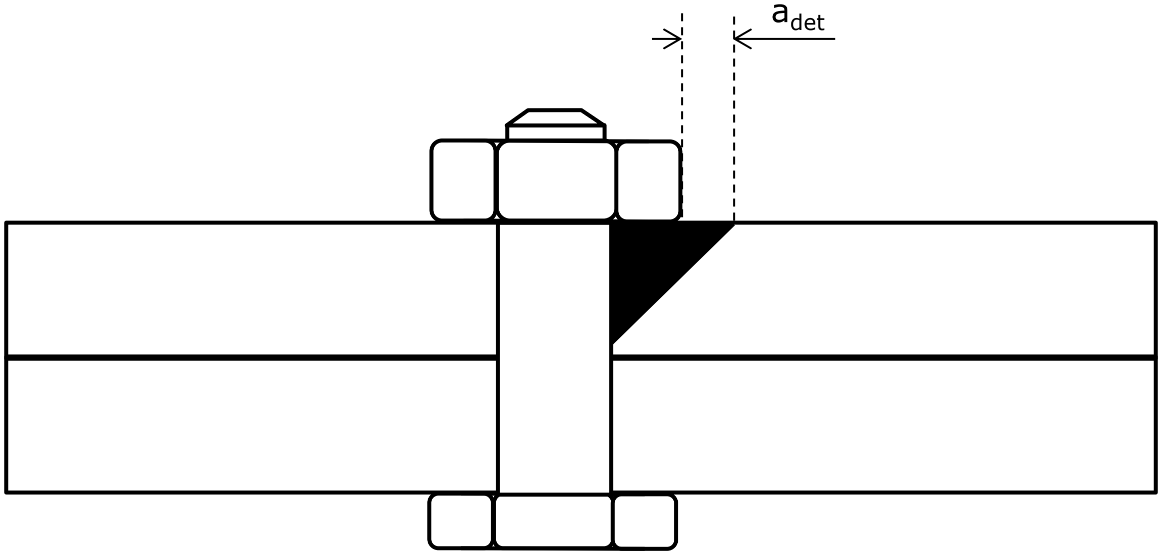
INSPECTION INTERVALS



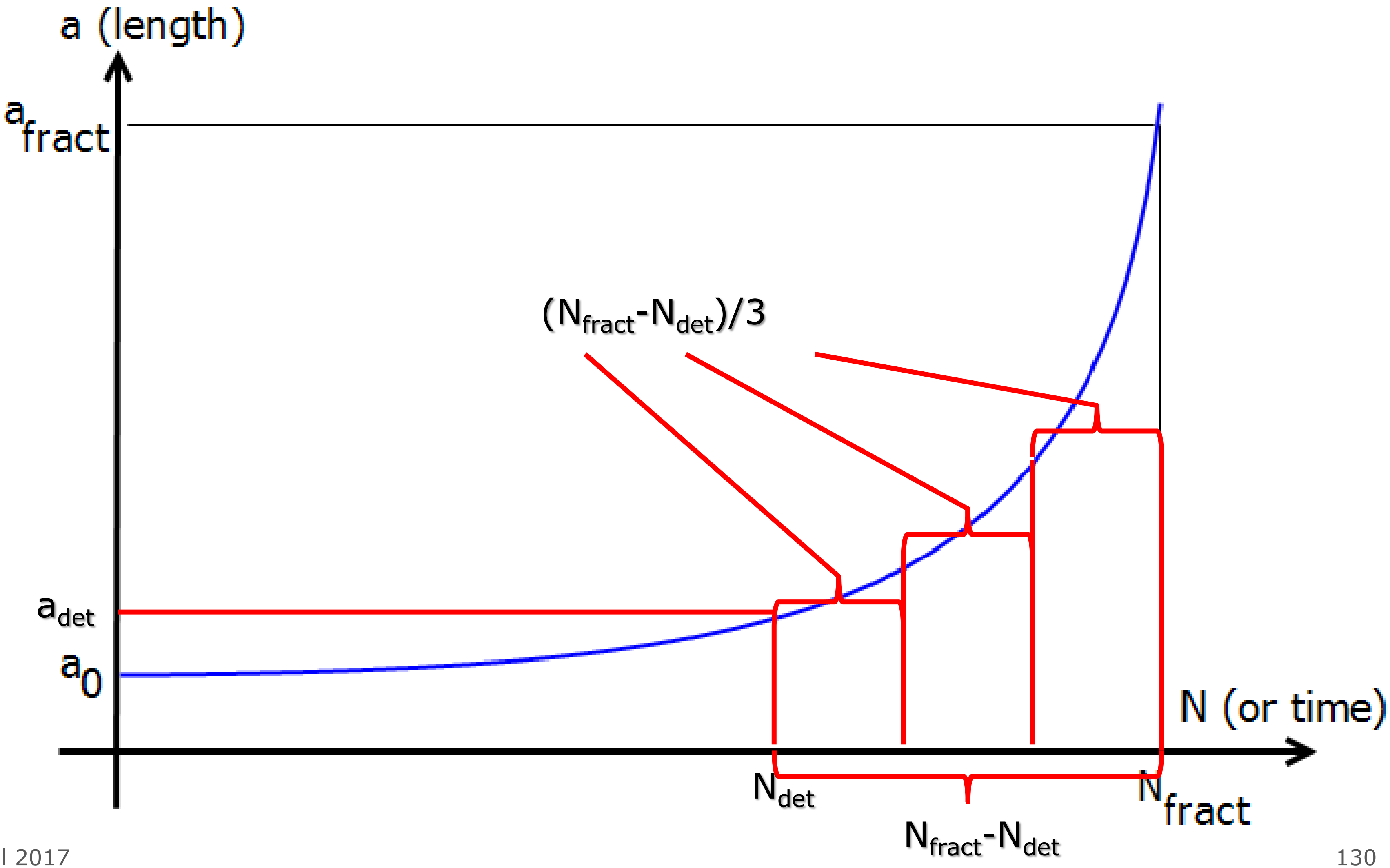
INSPECTION INTERVALS



INSPECTION INTERVALS



INSPECTION INTERVALS



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THANK YOU

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I-PTFC
We will miss you Professore Pascale!!!